

Finite element analysis of thermal stress of laminated composite plates using Taguchi method

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ABSTRACT

In present study, thermal stress behavior of laminated composite plates subjected to constant temperature load was investigated using Taguchi method according to von Mises stress. Numerical thermal stress analyses were conducted based on L8 orthogonal array including four control factors with two levels. Fiber orientation angles of laminates for the plates were used to be control factors. Finite element analysis of the plates was carried out using ANSYS software. Fiber angles with optimum levels and their effects on thermal stress of plates were detected using analysis of signal to noise (S/N) ratio. Level of importance of laminates and percentage effect ratio of each laminate on von Mises stress were determined according to analysis of variance (ANOVA).

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1. Introduction

Laminated composite plates can be used in different application areas of engineering. The plates made of laminated composite materials have some superior properties compared to metal and ceramic materials. Therefore the need for these materials is increasing. In addition, the increase in temperature can limit the use characteristics of laminated composite plates. Because of that, thermal stress behavior of these plates is important. Finite element approach was used in some of these analyses [1]. In literature, there are many studies about thermal stress of laminated composite plates. Thangaratnam et al. [2] investigated the thermal stress behavior of plates and shells made from laminated composite. Savoia and Reddy [3] evaluated the thermal behavior of laminated composite plates subjected to thermal load according to three dimensional. Tungikar and Rao [4] reported an exact solution stress of rectangular composite laminate subjected to temperature load according to three dimensional approach. Sayyad et al. [5] analyzed of thermal stress of plates made of laminated composite based on exponential shear deformation approach. Bektas and Sayman [6] presented the analytical approach about elasto-plastic stress of thermoplastic laminated plates under simply supported boundary condition. Sit et al. [7] examined the thermal stress behavior of plates made from laminated composite based on third order shear deformation approach. Wu et al. [8] investigated the thermal stress of plates made from laminates

according to actual temperature field. Khdeir and Reddy [9] analyzed the thermal stress and deflection behaviors of plates made from cross-ply laminates according to refined plate methods. Kant and Khare [10] presented a study about finite element approach of thermal stress of composite laminates based on a higher-order technique. Raju and Kumar [11] carried out the thermal behavior of plates made from laminated composite according to higher-order shear deformation approach which having zig-zag function. In this study, von Mises stress analysis of laminated composite plates subjected to temperature load was determined using finite element and Taguchi methods. Numerical analyses were conducted using finite element software ANSYS. Tests were performed using L8 orthogonal array based on Taguchi method. Each laminate groups has two laminates and their fiber orientation angles were assumed to be control parameters.

2. Materials and methods

The square laminated composite plate has eight laminates and it were made from graphite/epoxy materials. The material constants were given in Table 1.

Table 1. Material constants [12-14]

E_1	$E_2 = E_3$	$G_{12} = G_{13}$	G_{23}	$\nu_{12} = \nu_{13}$	ν_{23}	α_1	$\alpha_2 = \alpha_3$
155 GPa	8.07 GPa	4.55 GPa	3.25 GPa	0.22	0.373	$-0.07 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$	$30.1 \times 10^{-6} \text{ } ^\circ\text{C}^{-1}$

The planar dimension of plates made of laminated composite and thickness of each laminate were determined to be 150 mm and 0.125 mm [12] respectively. Thermal stress analysis of laminated composite plates were conducted using L8 orthogonal array based on Taguchi method. The orthogonal array consists of four control factors with two levels. Fiber orientation angles of composite laminates were considered to be control factors and each control factor was assumed to be two laminates. Eight analyses were performed numerically and statically. The control parameters with different levels were tabled in Table 2.

Table 2. Control factors with different levels

Control Factors	Symbol	Unit	Levels
First Two Laminates	A	Degree	0 10
Second Two Laminates	B	Degree	20 30
Third Two Laminates	C	Degree	40 50
Fourth Two Laminates	D	Degree	60 70

In order to obtain the optimum levels of fiber orientation angles of each laminate groups according to the maximum thermal stress for von Mises stress, “higher is better” quality characteristic was used. Analysis of S/N ratio was carried out

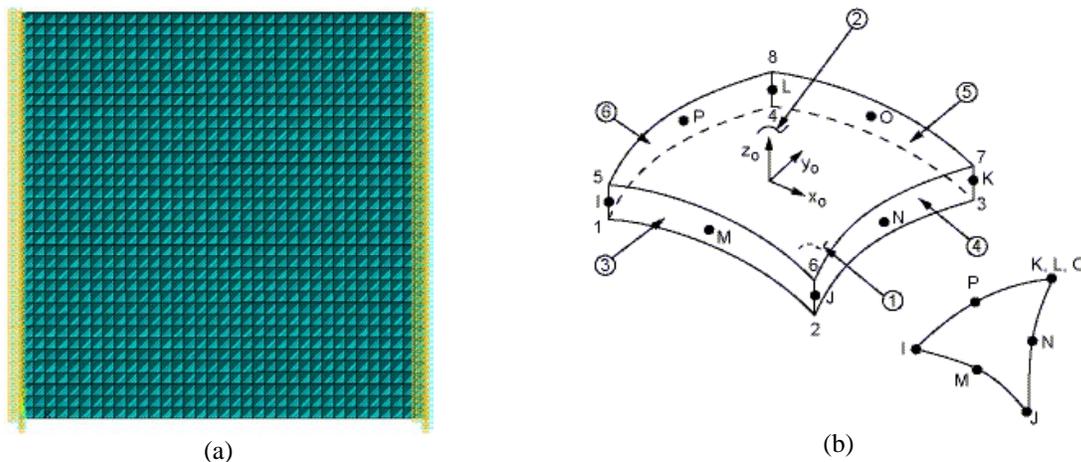
using Minitab R15 [15] statistical software. The quality characteristic was given in Equation 1 [16].

$$(S/N)_{HB} \text{ for } \sigma_T = -10 \cdot \log \left(n^{-1} \sum_{i=1}^n (y_i^2)^{-1} \right) \quad (1)$$

According to Equation 1, n is determined to be the number of thermal stress analyses based on a trial and y_i was used to be i th data analyzed.

3. Numerical approach

Finite element thermal stress analysis of laminated composite plates was analyzed using finite element software ANSYS [17]. Thermal stress results were determined for maximum data according to von Mises stress. Right and left edges of plates were assumed to be clamped boundary conditions whereas top and bottom edges of plates were considered to be free boundary conditions. Fiber orientation angles with 0 in degree were employed in x axis direction. Laminated composite plates were subjected to constant temperature load with $\Delta T = 70 \text{ } ^\circ\text{C}$ on surface area of the plates. Problem dimensionality was determined as three dimensional. Degrees of freedom was studied to be UX, UY, UZ, ROTX, ROTY, and ROTZ. Globally assembled matrix was taken to be symmetric. In finite element analyses, SHELL 281 element type [18] in modelling of plates was used. This element type includes eight nodes which having six degree of freedom according to each node: translations based on the x, y, and z axes, and rotations for the x, y, and z-axes [18]. The laminated composite plates with clamped-clamped boundary conditions of right and left edges and SHELL 281 geometry [18] were shown in fig. 1.

**Figure 1.** a) laminated composite plates and b) SHELL281 geometry [18]

4. Results and discussions

Numerical thermal stress behavior of laminated composite plates was conducted using finite element software ANSYS according to Taguchi's L8 orthogonal array. Finite element results for thermal stress and their data for signal to noise ratio were given in Table 3.

Table 3. Finite element and S/N ratio results

Test	Designation	Control Factors				Results	
		A	B	C	D	Von Mises Stress σ_T (MPa)	S/N Ratios η (dB)
1	A ₁ B ₁ C ₁ D ₁	0	20	40	60	310.873	49.8517
2	A ₁ B ₁ C ₂ D ₂	0	20	50	70	354.209	50.9852
3	A ₁ B ₂ C ₁ D ₂	0	30	40	70	320.124	50.1064
4	A ₁ B ₂ C ₂ D ₁	0	30	50	60	305.509	49.7005
5	A ₂ B ₁ C ₁ D ₂	10	20	40	70	405.287	52.1553
6	A ₂ B ₁ C ₂ D ₁	10	20	50	60	392.913	51.8859
7	A ₂ B ₂ C ₁ D ₁	10	30	40	60	358.235	51.0834
8	A ₂ B ₂ C ₂ D ₂	10	30	50	70	404.968	52.1484
Overall Mean (\bar{T}_{σ_T})						356.5148	-

4.1. Effect of Fiber Angles

In order to analyze of effect of fiber orientation angles of composite laminates on thermal stress, average results of each control factor for each level based on numerical and S/N ratio values of thermal stress were calculated. The average results for numerical and S/N ratio were tabulated in Table 4.

Table 4. Response Table for S/N ratio and thermal stress

Level	S/N ratios in dB				Means in MPa			
	A	B	C	D	A	B	C	D
1	50.16	51.22	50.8	50.63	322.7	365.8	348.6	341.9
2	51.82	50.76	51.18	51.35	390.4	347.2	364.4	371.1
Delta	1.66	0.46	0.38	0.72	67.7	18.6	15.8	29.3
Rank	1	3	4	2	1	3	4	2

According to Table 4, the control factors with the optimum levels were found to be A with second level, B with first level, C with second level, and D with second level. In order to see the effects of fiber orientation angles on the thermal stress, average results of S/N ratio for each levels of control factors were used. The average results were plotted in Fig. 2. It can be seen from Figure 2 that the increase of fiber orientation angles for the first two laminates, the third two laminates, and the fourth two laminates causes the increase of thermal stress of laminates composite plates. However, the increase of the fiber angles of the second two laminates provides the decrease of thermal stress of laminated composite plates.

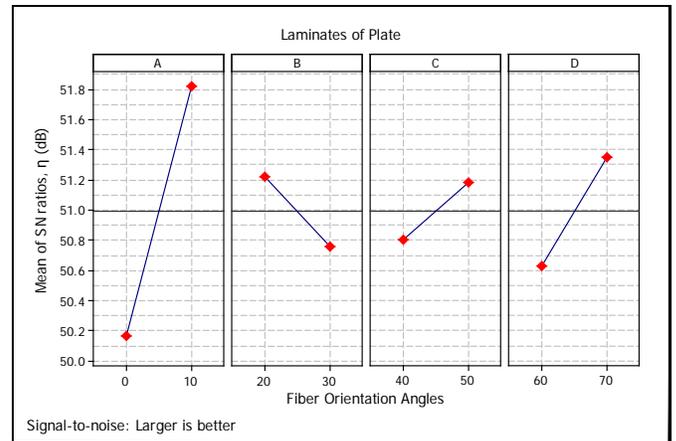


Figure 2. Effect of fiber orientation angles

4.2. Analysis of Variance

Laminated composite plates were designed to be eight laminates and fiber orientation angle of each two laminates was assumed to be control factor. In order to examine significance level and contribution ratio the fiber orientation angles of laminates of the plates, analysis of variance (ANOVA) was employed at 95 % confidence level using finite element results for thermal stress based on von Mises stress. The ANOVA result for R-Sq = 99.95 %, and R-Sq(adj) = 99.88 % was shown in Table 5.

Table 5. ANOVA result

Source	DF	Seq SS	Adj SS	Adj MS	F	P	% Effect
A	1	9159	9159	9159.0	4151.88	0.000	75.89
B	1	692.8	692.8	692.8	314.04	0.000	5.74
C	1	497.4	497.4	497.4	225.47	0.001	4.12
D	1	1712.8	1712.8	1712.8	776.44	0.000	14.19
Error	3	6.6	6.6	2.2			0.06
Total	7	12068.6					100

S = 1.48526, R-Sq = 99.95 %, and R-Sq(adj) = 99.87 %

As can be seen from ANOVA result, the most effective laminates groups on the thermal stress were obtained to be A with 75.89 % effect, D with 14.19 % effect, B with 5.74 % effect, and C with 4.12 effect, respectively. Each laminate groups was found to be the significance control parameter because P value was smaller than 0.05 data.

4.3. Estimation of Optimum Thermal Stress

The optimum result of thermal stress of laminated composite plates for the maximum value based on von Mises stress was only predicted considering the influence of the significant control factors. The optimum value of thermal stress was obtained using A with second level, B with first level, C with second level, and D with second level. The estimated mean of

thermal stress of laminated composite plates can be calculated using Equation 2 [16].

$$\mu_{\sigma_T} = \bar{A}_2 + \bar{B}_1 + \bar{C}_2 + \bar{D}_2 - 3\bar{T}_{\sigma_T} \quad (2)$$

where, $\bar{A}_2 = 390.4$, $\bar{C}_2 = 364.4$, and $\bar{D}_2 = 371.1$ are average data of numerical thermal stress at the second levels of laminate groups such as A, C, and D. These average values were taken from Table 4. In addition, $\bar{B}_1 = 365.8$ is average result of numerical thermal stress at the first level of laminate groups such as B. (\bar{T}_{σ_T}) = 356.5148 is overall mean according to L8 orthogonal array and this result was taken from Table 3. Substituting these values of different terms in Equation 2, μ_{σ_T} is calculated to be 422.156 MPa. 95 % confidence intervals of confirmation numerical thermal stress behavior and population were solved by using Equation 3 and Equation 4 [16] respectively.

$$CI_{CA} = \left(F_{\alpha;1;n_2} V_{error} \left[\frac{1}{n_{eff}} + \frac{1}{R} \right] \right)^{1/2} \quad (3)$$

$$CI_{POP} = \left(\frac{F_{\alpha;1;n_2} V_{error}}{n_{eff}} \right)^{1/2} \quad (4)$$

$$n_{eff} = \frac{N}{(1 + T_{DOF})} \quad (5)$$

where, $\alpha = 0.05$ is determined as risk and $n_2 = 3$ is analyzed as the error data for the degree of freedom in ANOVA. $F_{0.05;1;3}$ is employed as 10.13 [16] for F ratio table at 95 % CI. V_{error} is investigated to be the error result of variance based on ANOVA data and it is found to be 2.2 value. R is achieved as the sample size of confirmation analyses of numerical thermal stress and it is studied to be 1. N represents the total number of analyses of numerical thermal stress and it was evaluated to be 8 depending on Taguchi's L8 orthogonal array. T_{DOF} is analyzed as the total number of degrees of freedom according to the important control parameters and it was conducted to be 4. n_{eff} was solved to be 1.6 value so CI_{CA} and CI_{POP} were solved to be ± 6.018 and ± 3.732 respectively. The estimated confidence interval according to confirmation analyses for finite element thermal stress [16] is:

$$\text{Mean } \mu_{\sigma_T} - CI_{CA} < \mu_{\sigma_T} < CI_{CA} + \text{Mean } \mu_{\sigma_T}$$

The population depending on the 95 % confidence interval [16] is:

$$\text{Mean } \mu_{\sigma_T} - CI_{POP} < \mu_{\sigma_T} < CI_{POP} + \text{Mean } \mu_{\sigma_T}$$

The estimated and finite element optimal results depending on predicted confidence intervals were tabulated in Table 6.

Table 6. Optimal results for numerical and estimated data

Designation	Numerical Result	Predictive Result	Estimated Confidence Intervals for 95% Confidence Level
A ₂ B ₁ C ₂ D ₂	423.301	422.156	416.138 < μ_{σ_T} < 428.174 for CI _{CA}
	MPa	MPa	418.424 < μ_{σ_T} < 425.888 for CI _{POP}

5. Conclusion

In this study, von Mises stress of laminated composite plates subjected to constant temperature load was investigated by using finite element software ANSYS and Taguchi method. Fiber orientation angles of laminates of the plates were designed based on L8 orthogonal array consisting of four control factors with two levels. The optimal fiber angles and their effects were investigated using analysis of signal to noise ratio. ANOVA was used in order to analyze the significant laminate groups of composite plates and their percentage effects on the thermal stress. The following conclusions were drawn for this numerical and statistical study:

- Stress based on von Mises was found to be 356.5148 MPa according to L8 orthogonal array.
- Thee increase of fiber orientation angles for the first two laminates, the third two laminates, and the fourth two laminates leads to the increase of thermal stress of laminates composite plates whereas the increase of the fiber angles of the second two laminates causes the decrease of thermal stress.
- The optimum laminate groups were determined to be A₂B₁C₂D₂ according to Taguchi method.
- According to ANOVA, the most effective laminates groups on the thermal stress were found to be A with 75.89 % effect, D with 14.19 % effect, B with 5.74 % effect, and C with 4.12 effect, respectively.
- All of the laminate groups were determined as significant control parameters since P value in ANOVA was smaller than 0.05 data.
- Estimated confidence intervals for 95% confidence level were carried out to be 416.138 < μ_{σ_T} < 428.174 for CI_{CA} and 418.424 < μ_{σ_T} < 425.888 for CI_{POP}.
- Numerical and estimated optimum thermal stress results were solved to be 423.301 MPa and 422.156 MPa respectively.

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