Aksaray University
Journal of Science and Engineering

e-ISSN: 2587-1277
http://asujse.aksaray.edu.tr

Aksaray J. Sci. Eng.
Volume **9,** Issue **1,** pp. **23-34.**

Available online at DergiPark

Research Article

Optimization of Concrete Based on Scheffe's Model Using Crushed Glass as Partial Replacement for Fine Aggregate

Bala Alhaji Abbas[1,*], Nurudeen Adisa Sulaimon[1], Jibrin Abubakar[2], Uthman Hamidu Jantabo[1], Sodiq Alabi Alao[1], Israel Oluwayomi Akere[1]

[1]*Department of Civil Engineering, Federal University of Technology, Minna, Nigeria.*
[2]*Department of Civil Engineering, Joseph Sarwuan Tarka University, Makurdi, Nigeria.*

Keywords
Crushed glass, Fine aggregate, Scheffe's optimization model, Simplex design, Compressive strength

Article information
Received: Jul 23, 2024
Revised: Dec 16, 2024
Accepted: Mar 14, 2025
Online: Apr 24, 2025

doi: **10.29002/asujse.1520782**

Abstract
This study developed a mathematical model for optimizing the 28-day compressive strength of concrete containing crushed glass and Bida Natural Gravel, based on Scheffe's simplex theory. Using a total of 30 mix ratios, 90 concrete cubes were cast to validate the model. Fifteen mix ratios derived from simplex lattice points were used to calculate model coefficients, while the remaining 15 pseudo-random mixes served as control points. The model's predictions were statistically validated using the Fischer test, demonstrating adequacy with a 95 percent confidence level. Optimization via MATLAB revealed an ideal 28-day compressive strength of 36.83 MPa, corresponding to an optimal mix ratio—0.4739:1:0.6854:0.1952:1.7612 for water, cement, sand, crushed glass, and BNG. This model enables prediction of mix ratios for desired strengths of concrete containing 0 percent to 25 percent crushed glass.

1. Introduction

Recycling industrial waste and byproducts, including broken glass, has advanced significantly in the building sector. In addition to saving landfill space, recycling this material by turning it into aggregate also lessens the need for natural raw materials for construction work [1]. Numerous research investigations have been conducted since these alternatives necessitate in-depth analysis of their impact on concrete's properties. According to Park *et al*. [2], the compressive, tensile, and flexural strengths of concrete using crushed glass as fine aggregate showed a tendency to decline with increases in the crushed glass' mixing ratio.

Shayan and Xu [3] discovered that 30% glass powder could be used as an alternative to cement or aggregate in concrete without having a negative long-term impact. No alkali-silica reaction has been observed with particle sizes up to 100 microns, according to Corinaldesi *et al*. [4], supporting the viability of crushed glass as fine aggregate in mortar and concrete. In late ages, crushed glass concrete mixes' compressive strength significantly increased, according to Chen *et al*. [5]. The addition of finely milled glass in concrete mixes, according to Metwally [6], had a negative impact on workability but significantly improved the mechanical characteristics of concrete over time.

In determining a specified or desired compressive strength of a cement-based composite material, achieving the right proportioning of concrete mix is very important [7]. Chen *et al*. [8] demonstrated the use of the simplex lattice design method for forecasting cement-based composites' characteristics. Their demonstration used ternary systems made of cement, silica fume, and fly ash with a consistent water-cement ratio to demonstrate compressive strength. Additionally, Nwakonobi and Osadebe [9] modelled a useful optimization function to predict the compressive strength of a quaternary system using the simplex-lattice design. Their model had four components namely clay, rice husk, cement, and water. Anyaogu and Ezeh [10] used Scheffe's simplex theory to model and optimize the compressive strength of concrete made using normal aggregates and fly ash blended cement. According to their research, the model that was developed can be used to estimate mix ratios for fly ash blended cement concrete of any required strength within the (5,2) factor space of a simplex model. Additionally, Simon [11]; Ezeh and Ibearugbulem [12]; Osadebe [13] illustrated the use of mathematical modelling in civil engineering in their various publications.

Corresponding author: bala.alhaji@futminna.edu.ng 0000-0002-1322-093X

Structural engineers often utilize statistical experimental design approaches in concrete mixture proportioning by developing design which is best feasible in terms of cost, weight, dependability or a combination of these parameters [14]. According to Kalantari *et al.* [15], choosing the right mix proportion is crucial for choosing the appropriate materials needed to produce concrete as well as for maximizing crucial parameters including compressive strength, durability, and smooth consistency. This study leverages this theory to model and optimize the compressive strength of concrete mixes with crushed glass as a partial replacement for fine aggregate.

2. Materials and Methods

2.1. Materials

Materials used for this research work are sand, cement, Bida gravel, water and waste glass. Ordinary Portland Cement grade 42.5N (Normal hardening and 28-day compressive strength of 42.5N/mm^2) was used for this research. Fine aggregate was sourced from Minna, Niger state, Nigeria. Bida gravel was gotten from Bida, Niger state as shown in Fig. 1. The gravel was washed in 5mm British Standard sieve to remove clay impurities which may affect concrete production and dried. Portable water was gotten from the civil engineering laboratory. The water used was colourless, odourless and free from visible impurities. Crushed glass was obtained from discarded glass bottles at the central workshop, mechanical engineering department, Federal University of Technology, Minna and processed by cleaning, crushing, and sieving into specific particle size ranges. Fig. 2. shows the crushed glass. Only particles passing sieve size 1.18 mm British standard sieve were used in concrete as partial replacement for sand. No alkali-silica reaction was found with particle sizes up to 100 µm, according to Corinaldesi *et al*. [4]. Its granulometric distribution was controlled to ensure compatibility with fine aggregates, and the fine material content was approximately 15%, contributing to improved packing density and reduced voids in the concrete mix. The Cu and Cc of the crushed glass are 6 and 1.22 respectively, and the crushed glass is well-graded. From Table 1, the specific gravity of the crushed glass is 2.51. Hence, it is classified as a normal weight aggregate.

Table 1. Physical properties of the aggregate

Physical Properties	Materials		
	Sand	CG	BNG
Fineness Modulus	2.7	2.5	6.4
Absorption (%)	2.68	2.60	2.37
Specific gravity	2.6	2.51	2.68
Density (kg/m^3)	1515	1453	1663
AIV (%)	-	-	16.56

Figure 1. Sample of Bida Natural Gravel (BNG)

Figure. 2. Sample of crushed glass

2.2. Concrete Mix

A total of 90 concrete cubes (150×150×150 mm) were cast. The experimental mixes comprised 15 simplex lattice-derived and 15 pseudo-random control points, as shown in Tables 2 and 3. The Scheffe optimization equation was developed and validated using compressive strength data obtained at 28 days, with statistical adequacy confirmed via the Fischer test.

2.3. Simplex Design

The features of a simplex lattice design comprising q components and m degree polynomial include symmetric arrangement of points within the experimental zone and a polynomial function of the response over the simplex region. The corresponding simplex lattice design's number of points is exactly matched by the number of parameters in the polynomial. Experimental points in a (q, m) simplex lattice as designed by Scheffe in 1958 is given by ${}^{q+m-1}C_m$ points. q is the number of components in concrete mix and m is the degree of the Scheffe's optimization equation. Each component takes (m + 1) equally spaced value.

$$x_i = 0, \frac{1}{m}, \frac{2}{m}, \ldots \ldots \ldots \ldots, 1 \tag{1}$$

Where i = 1,2, …, q ranges between 0 and 1. For (5, 2) simplex lattice, it can be written in the form ${}^{5+2-1}C_2 = {}^{6}C_2 = 15$ points. x_i = 0, $\frac{1}{2}$, 1 with which possible design pseudo mix ratios are as presented in Fig. 3.

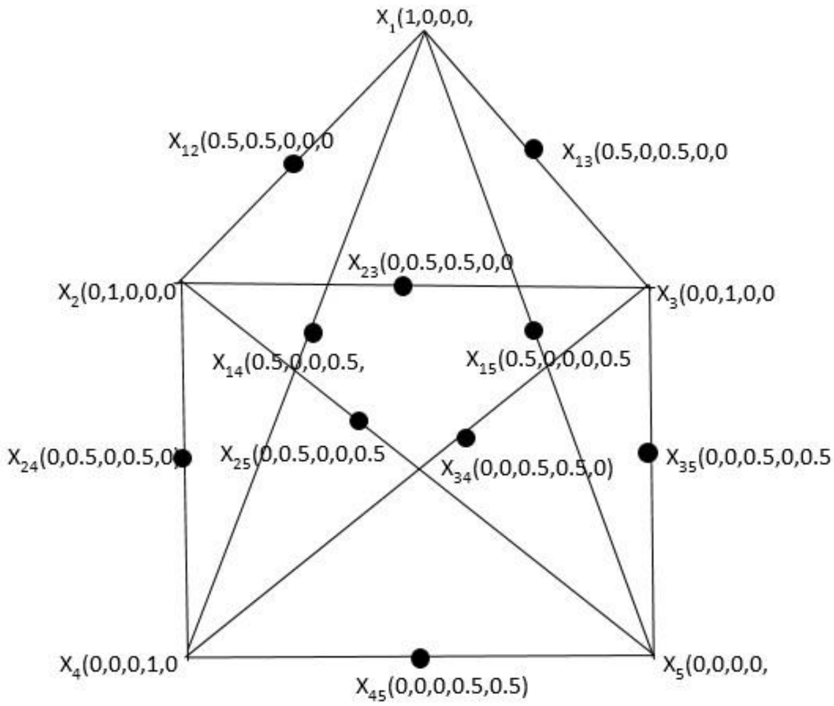

Figure 3. A (5, 2) Scheffe's Simplex Lattice

The pseudo mix ratios in Fig. 3. were used to generate actual mix ratios from Equation 2. To validate the model, extra 15 actual mix ratios (control) were determined from Equation 2 by randomly generation additional 15 pseudo mix ratios. The strength attained at 28 days from the control mix ratios were used in the Fischer-statistical test. The statistical test was done to ascertain whether the difference between the experimental and model results was significant or not.

$$[S] = [A]^T [X]^T \tag{2}$$

S, A and X are the actual mix ratio, coefficient of relation matrix, and pseudo mix ratio respectively.

2.4. Scheffe's Optimization Equation

According to Scheffe [16] and Simon *et al*. [17], the characteristics of freshly mixed and hardened concrete are referred to as responses, and they can be modelled as a polynomial function of a pseudo component of the mixture as shown below.

$$Y = b_o + \sum_{i=1}^{q} b_i X_i + \sum_{i,j=1}^{q} b_{ij} X_i X_j + \sum_{i,j,k=1}^{q} b_{ijk} X_i X_j X_k + \cdots \ldots \ldots . . X_{in} + e \tag{3}$$

b_o = arbitrary constant, e = random error and Y is the response.

The response equation for five pseudo component mixture (cement, sand, crushed glass, BNG and water) can be written as:

$$\begin{aligned} Y = b_o &+ b_1 X_1 + b_2 X_2 + b_3 X_3 + b_4 X_4 + b_5 X_5 + b_{11} X_1^2 + b_{12} X_1 X_2 + b_{13} X_1 X_3 + b_{14} X_1 X_4 + b_{15} X_1 X_5 \\ &+ b_{22} X_2^2 + b_{23} X_2 X_3 + b_{24} X_2 X_4 + b_{25} x_2 X_5 + b_{33} x_3^2 + b_{34} X_3 X_4 + b_{35} X_3 X_5 + b_{44} X_4^2 \\ &+ b_{45} X_4 X_5 + b_{55} X_5^2 + e \end{aligned} \tag{4}$$

The term e which is the random error can be neglected.

$$\sum_{i=1}^{q} X_i = 1 \tag{5}$$

Where q is 5, Equation 5 can be written as

$$\sum_{i=1}^{5} X_i = 1 \tag{6}$$

$$X_1 + X_2 + X_3 + X_4 + X_5 = 1 \tag{7}$$

Multiplying Equation 7 by b_o

$$b_o = b_o(X_1 + X_2 + X_3 + X_4 + X_5) \tag{8}$$

$$b_o = b_oX_1 + b_oX_2 + b_oX_3 + b_oX_4 + b_oX_5 \tag{9}$$

Multiplying equation (7) by X_1, X_2, X_3, X_4 and X_5 and making X_i^2 the subject of the formulas

$$\begin{gathered} X_1^2 = X_1 - X_1X_2 - X_1X_3 - X_1X_4 - X_1X_5 \\ X_2^2 = X_2 - X_1X_2 - X_2X_3 - X_2X_4 - X_2X_5 \\ X_3^2 = X_3 - X_1X_3 - X_2X_3 - X_3X_4 - X_3X_5 \\ X_4^5 = X_4 - X_1X_4 - X_2X_4 - X_3X_4 - X_4X_5 \\ X_5^2 = X_5 - X_1X_5 - X_2X_5 - X_3X_5 - X_4X_5 \end{gathered} \tag{10}$$

Substituting equations (9) and (10): into equation (4)

$$\begin{aligned} Y = {} & b_oX_1 + b_oX_2 + b_oX_3 + b_oX_4 + b_oX_5 + b_1X_1 + b_2X_2 + b_3X_3 + b_4X_4 + b_5X_5 + b_{11}X_1 - b_{11}X_1X_2 \\ & - b_{11}X_1X_3 - b_{11}X_1X_4 - b_{11}X_1X_5 + b_{12}X_1X_2 + b_{13}X_1X_3 + b_{14}X_1X_4 + b_{15}X_1X_5 + b_{22}X_2 \\ & - b_{22}X_1X_2 - b_{22}X_2X_3 - b_{22}X_2X_4 - b_{22}X_2X_5 + b_{23}X_2X_3 + b_{24}X_2X_4 + b_{25}X_2X_5 + b_{33}X_3 \\ & - b_{33}X_1X_3 - b_{33}X_2X_3 - b_{33}X_3X_4 - b_{33}X_3X_5 + b_{34}X_3X_4 + b_{35}X_3X_5 + b_{44}X_4 - b_{44}X_1X_4 \\ & - b_{44}X_2X_4 - b_{44}X_3X_4 - b_{44}X_4X_5 + b_{45}x_4X_5 + b_{55}X_5 - b_{55}X_1X_5 - b_{55}X_2X_5 - b_{55}X_3X_5 \\ & - b_{55}X_4X_5 \end{aligned} \tag{11}$$

Further simplifying equation (11)

$$\begin{aligned} Y = {} & X_1(b_o + b_1 + b_{11}) + X_2(b_o + b_2 + b_{22}) + X_3(b_o + b_3 + b_{33}) + X_4(b_o + b_4 + b_{44}) + X_5(b_o + b_5 + b_{55}) \\ & + X_1X_2(b_{12} - b_{11} - b_{22}) + X_1X_3(b_{13} - b_{11} - b_{33}) + X_1X_4(b_{14} - b_{11} - b_{44}) \\ & + X_1X_5(b_{15} - b_{11} - b_{55}) + X_2X_3(b_{23} - b_{22} - b_{33}) + X_2X_4(b_{24} - b_{22} - b_{44}) \\ & + X_2X_5(b_{25} - b_{22} - b_{55}) + X_3X_4(b_{34} - b_{33} - b_{44}) + X_3X_5(b_{35} - b_{44} - b_{55}) \\ & + X_4X_5(b_{45} - b_{44} - b_{55}) \end{aligned} \tag{12}$$

The constants in parenthesis can sum up to give another constant say β and let

$$\begin{gathered} \beta_1 = b_o + b_1 + b_{11} \\ \beta_2 = b_o + b_2 + b_{22} \\ \beta_3 = b_o + b_3 + b_{33} \\ \beta_4 = b_o + b_4 + b_{44} \\ \beta_5 = b_o + b_5 + b_{55} \\ \beta_{12} = b_{12} - b_{11} - b_{22} \\ \beta_{13} = b_{13} - b_{11} - b_{33} \\ \beta_{14} = b_{14} - b_{11} - b_{44} \\ \beta_{15} = b_{15} - b_{11} - b_{55} \\ \beta_{23} = b_{23} - b_{22} - b_{33} \\ \beta_{24} = b_{24} - b_{22} - b_{44} \\ \beta_{25} = b_{25} - b_{22} - b_{55} \\ \beta_{34} = b_{34} - b_{33} - b_{44} \end{gathered} \tag{13}$$

$$\beta_{35} = b_{35} - b_{33} - b_{55}$$

$$\beta_{45} = b_{45} - b_{44} - b_{55}$$

Substituting equation (13) into equation (12)

$$Y = X_1\beta_1 + X_2\beta_2 + X_3\beta_3 + X_4\beta_4 + X_5\beta_5 + X_1X_2\beta_{12} + X_1X_3\beta_{13} + X_1X_4\beta_{14} + X_1X_5\beta_{15} + X_2X_3\beta_{23} + X_2X_4\beta_{24} + X_2X_5\beta_{25} + X_3X_4\beta_{34} + X_3X_5\beta_{35} + X_4X_5\beta_{45} \quad (14)$$

Equation (14) can be written as

$$Y = \sum_{i=1}^{5} X_i\beta_i + \sum_{1\le i\le j\le 5} X_iX_j\beta_{ij} \quad (15)$$

Let's designate n_i is the response to pure components and n_{ij} is the response to mixed components

$$\beta_i = n_i \quad (16)$$

$$\sum_{i=1}^{5} X_i\beta_i = \sum_{i=1}^{5} X_i n_i \quad (17)$$

In a similar manner, the values of responses for the midpoints between X_1, X_2, X_3, X_4 and X_5 was

$$n_{ij} = \frac{1}{2}\beta_i + \frac{1}{2}\beta_j + \frac{1}{4}\beta_{ij} \quad (18)$$

$$4n_{ij} = 2\beta_i + 2\beta_j + \beta_{ij} \quad (19)$$

$$\beta_{ij} = 4n_{ij} - 2\beta_i - 2\beta_j \quad (20)$$

Substituting equations (18), (19) and (20): into equation (14)

$$Y = X_1n_1 + X_2n_2 + X_3n_3 + X_4n_4 + X_5n_5 + X_1X_2(4n_{12} - 2n_1 - 2n_2) + X_1X_3(4n_{13} - 2n_1 - 2n_3) + X_1X_4(4n_{14} - 2n_1 - 2n_4) + X_1X_5(4n_{15} - 2n_1 - 2n_5) + X_2X_3(4n_{23} - 2n_2 - 2n_3) + X_2X_4(4n_{24} - 2n_2 - 2n_4) + X_2X_5(4n_{25} - 2n_2 - 2n_5) + X_3X_4(4n_{34} - 2n_3 - 2n_4) + X_3X_5(4n_{35} - 2n_3 - 2n_5) + X_4X_5(4n_{45} - 2n_4 - 2n_5) \quad (21)$$

Further Simplifying equation (21)

$$Y = X_1n_1(1 - 2X_2 - 2X_3 - 2X_4 - 2X_5) + X_2n_2(1 - 2X_1 - 2X_3 - 2X_4 - 2X_5) + X_3n_3(1 - 2X_1 - 2X_2 - 2X_4 - 2X_5) + X_4n_4(1 - 2X_1 - 2X_2 - 2X_3 - 2X_5) + X_5n_5(1 - 2X_1 - 2X_2 - 2X_3 - 2X_4) + 4X_1X_2n_{12} + 4X_1X_3n_{13} + 4X_1X_4n_{14} + 4X_1X_5n_{15} + 4X_2X_3n_{23} + 4X_2X_4n_{24} + 4X_2X_5n_{25} + 4X_3X_4n_{34} + 4X_3X_5n_{35} + 4X_4X_5n_{45} \quad (22)$$

Recall from equation (7)

$$X_1 + X_2 + X_3 + X_4 + X_5 = 1$$

Multiply equation (7) by 2

$$2X_1 + 2X_2 + 2X_3 + 2X_4 + 2X_5 = 2 \quad (23)$$

Subtracting 1 from equation (23) (both RHS and LHS)

$$2X_1 + 2X_2 + 2X_3 + 2X_4 + 2X_5 - 1 = 1 \quad (24)$$

Rearranging equation (24) gives

$$2X_1 - 1 = 1 - 2X_2 + 2X_3 + 2X_4 + 2X_5 \quad (25)$$

Similarly

$$2X_2 - 1 = 1 - 2X_1 + 2X_3 + 2X_4 + 2X_5$$

$$2X_3 - 1 = 1 - 2X_1 + 2X_2 + 2X_4 + 2X_5$$

$$2X_4 - 1 = 1 - 2X_1 + 2X_2 + 2X_3 + 2X_5$$

$$2X_5 - 1 = 1 - 2X_1 + 2X_2 + 2X_3 + 2X_4 \quad (26)$$

Substituting equations (25) and (26): into equation (22)

$$Y = X_1 n_1 (2X_1 - 1) + X_2 n_2 (2X_2 - 1) + X_3 n_3 (2X_3 - 1) + X_4 n_4 (2X_4 - 1) + X_5 n_5 (2X_5 - 1) + 4X_1 X_2 n_{12} + 4X_1 X_3 n_{13} + 4X_1 X_4 n_{14} + 4X_1 X_5 n_{15} + 4X_2 X_3 n_{23} + 4X_2 X_4 n_{24} + 4X_2 X_5 n_{25} + 4X_3 X_4 n_{34} + 4X_3 X_5 n_{35} + 4X_4 X_5 n_{45} \quad (27)$$

Equation (27) gives the Scheffe's optimization equation for the mixture.

3. Results

3.1. Generating Actual and Pseudo Mixes from Lattice and Control Points

Fifteen (15) simplex lattice points generated from Fig. 3. and fifteen (15) randomly generated control points are thus referred to as pseudo mix ratios from which the actual mixes were generated as given by Equations 2 and 28. Table 2 presents the mix proportions for the simplex lattice points.

$$[S] = [A]^T [X]^T$$

Likewise, for the control points

$$[Z] = [A]^T [X]^T \quad (28)$$

$$A = \begin{pmatrix} 0.65 & 1 & 2.85 & 0.15 & 6 \\ 0.55 & 1 & 1.8 & 0.2 & 4 \\ 0.6 & 1 & 1.28 & 0.22 & 3 \\ 0.45 & 1 & 0.8 & 0.2 & 2 \\ 0.5 & 1 & 0.56 & 0.19 & 1.5 \end{pmatrix}$$

Table 2. Mix Proportions (Actual and Pseudo Components) for Simplex Lattice Points

Points	Pseudo Mix Ratios					Actual Mix Ratios				
	Water X_1	Cement X_2	Sand X_3	Glass X_4	BNG X_5	Water S_1	Cement S_2	Sand S_3	Glass S_4	BNG S_5
N_1	1	0	0	0	0	0.65	1.00	1.90	0.10	4.00
N_2	0	1	0	0	0	0.55	1.00	0.90	0.10	2.00
N_3	0	0	1	0	0	0.60	1.00	1.28	0.23	3.00
N_4	0	0	0	1	0	0.45	1.00	1.60	0.40	4.00
N_5	0	0	0	0	1	0.50	1.00	1.25	0.25	3.00
N_{12}	0.5	0.5	0	0	0	0.60	1.00	1.40	0.10	3.00
N_{13}	0.5	0	0.5	0	0	0.63	1.00	1.59	0.16	3.50
N_{14}	0.5	0	0	0.5	0	0.55	1.00	1.75	0.25	4.00
N_{15}	0.5	0	0	0	0.5	0.58	1.00	1.58	0.18	3.50
N_{23}	0	0.5	0.5	0	0	0.58	1.00	1.09	0.16	2.50
N_{24}	0	0.5	0	0.5	0	0.50	1.00	1.25	0.25	3.00
N_{25}	0	0.5	0	0	0.5	0.53	1.00	1.08	0.18	2.50
N_{34}	0	0	0.5	0.5	0	0.53	1.00	1.44	0.31	3.50
N_{35}	0	0	0.5	0	0.5	0.55	1.00	1.26	0.24	3.00
N_{45}	0	0	0	0.5	0.5	0.48	1.00	1.43	0.33	3.50

The Scheffe mix model was validated by generating additional fifteen (15) control points as shown in Table 3. For the concrete mixes used in this study, these ratios were used as the control mix ratios.

Table 3. Mix Proportions (Actual and Pseudo Components) for Control Points

Points	Pseudo Mix Ratios					Actual Mix Ratios				
	Water X_1	Cement X_2	Sand X_3	Glass X_4	BNG X_5	Water Z_1	Cement Z_2	Sand Z_3	Glass Z_4	BNG Z_5
C_1	0.5	0	0	0.25	0.25	0.56	1.00	1.66	0.21	3.75
C_2	0.25	0.5	0	0.25	0	0.55	1.00	1.33	0.18	3.00

C_3	0	0.25	0.25	0.5	0	0.51	1.00	1.34	0.28	3.25
C_4	0.25	0.25	0	0.5	0	0.53	1.00	1.50	0.25	3.50
C_5	0	0	0.5	0.25	0.25	0.54	1.00	1.35	0.28	3.25
C_6	0.33	0.33	0.33	0	0	0.59	0.99	1.34	0.14	2.97
C_7	0.33	0	0.33	0.33	0	0.56	0.99	1.58	0.24	3.63
C_8	0.33	0.33	0	0.33	0	0.54	0.99	1.45	0.20	3.30
C_9	0	0.33	0.33	0.33	0	0.53	0.99	1.25	0.24	2.97
C_{10}	0.33	0	0.33	0	0.33	0.58	0.99	1.46	0.19	3.30
C_{11}	0.25	0.25	0.25	0.25	0	0.56	1.00	1.42	0.21	3.25
C_{12}	0	0.25	0.25	0.25	0.25	0.53	1.00	1.26	0.24	3.00
C_{13}	0.25	0	0.25	0.25	0.25	0.55	1.00	1.51	0.24	3.50
C_{14}	0.25	0.25	0	0.25	0.25	0.54	1.00	1.41	0.21	3.25
C_{15}	0.25	0.25	0.25	0	0.25	0.58	1.00	1.33	0.17	3.00

3.2. Compressive strength

Table 4 presents the compressive strength of concrete cubes for the 30 experimental points at 28-day curing. The compressive strength of a concrete is the maximum axial compressive load it can withstand before failure. It is a measure of strength and a useful parameter in the structural design of concrete structures. The laboratory results presented in Table 4 are the parameters with which the Scheffe's model will be developed.

Table 4. 28-day Compressive Strength of Concrete

S/N	Water	Cement (kg)	Sand (kg)	Glass (kg)	BNG (kg)	Compressive Strength (N/mm^2)
1	2.20	3.38	6.91	0.36	15.48	16.71
2	3.05	5.55	5.37	0.60	12.71	16.03
3	2.52	4.20	5.76	1.02	14.43	23.44
4	1.61	3.58	6.17	1.54	16.41	26.67
5	2.18	4.35	5.86	1.17	14.96	24.67
6	2.52	4.20	6.33	0.45	14.43	25.69
7	2.34	3.74	6.40	0.65	15.01	21.13
8	1.91	3.48	6.55	0.94	15.93	23.78
9	2.19	3.80	6.45	0.72	15.25	17.24
10	2.75	4.78	5.60	0.84	13.69	21.48
11	2.18	4.35	5.86	1.17	14.96	23.76
12	2.56	4.88	5.64	0.92	13.97	22.99
13	2.03	3.87	5.98	1.30	15.50	19.16
14	2.35	4.27	5.81	1.09	14.69	39.63
15	1.87	3.93	6.03	1.37	15.76	36.81
16	2.04	3.63	6.50	0.83	15.61	19.29
17	2.35	4.27	6.10	0.81	14.69	22.79
18	2.10	4.09	5.92	1.24	15.25	23.04
19	2.03	3.87	6.24	1.04	15.50	22.21
20	2.18	4.06	5.90	1.20	15.12	24.07
21	2.50	4.21	6.15	0.64	14.46	20.33
22	2.07	3.69	6.33	0.96	15.52	24.74
23	2.17	3.98	6.28	0.86	15.19	24.64
24	2.27	4.31	5.84	1.12	14.81	20.87

25	2.27	3.93	6.25	0.81	15.03	21.04
26	2.26	4.03	6.15	0.89	14.99	20.73
27	2.26	4.31	5.83	1.13	14.83	23.85
28	2.11	3.83	6.22	1.01	15.38	22.30
29	2.18	4.06	6.17	0.93	15.12	24.30
30	2.44	4.24	6.07	0.77	14.56	26.27

3.3. Scheffe Mix Model

Table 4 provides the compressive strength at the 28-day curing age that was used in deriving the Scheffe mix model. Substituting the cube strength at the point of observations generated from the simple lattice structure into Equation 29, the Scheffe mix model is given as:

$$\begin{aligned} Y = {} & 16.71X_1(2X_1-1) + 16.03X_2(2X_2-1) + 23.44X_3(2X_3-1) + 26.67X_4(2X_4-1) \\ & + 24.67X_5(2X_5-1) + 102.76X_1X_2 + 84.52X_1X_3 + 95.12X_1X_4 + 68.96X_1X_5 \\ & + 85.92X_2X_3 + 95.04X_2X_4 + 91.96X_2X_5 + 76.64X_3X_4 + 158.52X_3X_5 + 147.24X_4X_5 \end{aligned} \tag{29}$$

Equation 29 is the mix model for the optimization of 28-day strength of concrete containing crushed glass and Bida natural gravel based on Scheffe's (5, 2) simplex lattice structure. The predicted 28-days compressive strengths of concrete cubes of the actual mix ratios generated from the simplex lattice and control points using Equation 29 are presented in Table 5.

Table 5. Experimental and Model 28-day Compressive Strength of Cubes

Points of Observation	Replica 1 (kN)	Replica 2 (kN)	Replica 3 (kN)	Mean Cube Strength (N/mm^2)	Predicted Cube Strength (N/mm^2)
n_1	520.00	202.00	406.00	16.71	16.71
n_2	248.00	432.00	402.00	16.03	16.03
n_3	422.00	615.00	545.00	23.44	23.44
n_4	585.00	560.00	655.00	26.67	26.67
n_5	555.00	600.00	510.00	24.67	24.67
n_{12}	630.00	650.00	454.00	25.69	25.69
n_{13}	490.00	436.00	500.00	21.13	21.13
n_{14}	550.00	515.00	540.00	23.78	23.78
n_{15}	304.00	460.00	400.00	17.24	17.24
n_{23}	530.00	350.00	570.00	21.48	21.48
n_{24}	555.00	414.00	635.00	23.76	23.76
n_{25}	600.00	407.00	545.00	22.99	22.99
n_{34}	453.00	412.00	428.00	19.16	19.16
n_{35}	900.00	825.00	950.00	39.63	39.63
n45	925.00	700.00	860.00	36.81	36.81
C_1	570.00	370.00	362.00	19.29	23.30
C_2	410.00	660.00	468.00	22.79	25.25
C_3	460.00	510.00	585.00	23.04	21.90
C_4	595.00	454.00	450.00	22.21	26.10
C_5	515.00	520.00	590.00	24.07	32.18

C_6	392.00	510.00	470.00	20.33	23.45
C_7	565.00	655.00	450.00	24.74	20.41
C_8	430.00	615.00	618.00	24.64	25.23
C_9	525.00	434.00	450.00	20.87	20.63
C_{10}	494.00	426.00	500.00	21.04	26.70
C_{11}	330.00	585.00	484.00	20.73	23.39
C_{12}	550.00	600.00	460.00	23.85	29.61
C_{13}	470.00	485.00	550.00	22.30	28.00
C_{14}	560.00	490.00	590.00	24.30	27.06
C_{15}	483.00	630.00	660.00	26.27	26.93

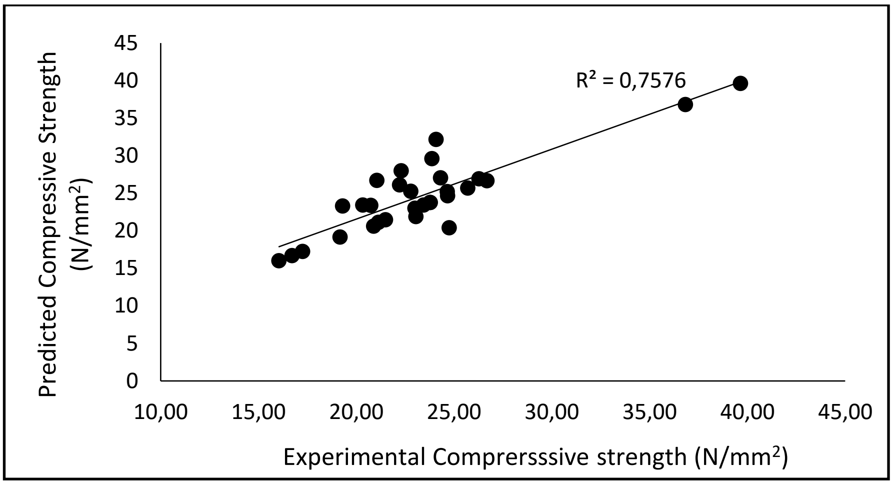

Figure 4. Regression model of experimental and predicted compressive strength of concrete

3.4. Test for Adequacy of the Scheffe Mix Model

A validity test of the mix model was carried out using the Fischer-statistical test at a 95% confidence level in the Microsoft Excel data analysis toolbox. The data set were the mean cube strength from laboratory test and predicted cube strength of concrete obtained from mix ratios generated from the control points C_1 - C_{15}. The null hypothesis, which states that there is no significant difference between the laboratory concrete cube strength and model predicted strength results, and the alternative hypothesis, which states there is a significant difference between the laboratory concrete cube strength and model, were the two hypotheses set for this test. If $P > 0.05$, the null hypothesis is accepted and the Scheffe mix model is suitable; otherwise, the alternate hypothesis is accepted. The outcome of the statistical test performed on the model is shown in Table 6.

Table 6. Fischer Statistical Test of Two Samples for Variances

	Predicted Cube Strength (N/mm^2)	Mean Cube Strength (N/mm^2)
Mean	24.6443312	23.32098765
Variance	27.072745	23.67491541
Observations	30	30
df	29	29
F	1.143520242	
P(F<=f) one-tail	0.360218706	
F Critical one-tail	1.860811435	

Since P is 0.36 and is greater than 0.05 and F-value of 1.14 is lesser than the $F_{crirtical}$ value of 1.86, the null hypothesis is accepted. It follows that the model is valid.

3.5. Optimization of Compressive Strength

The 28-day compressive strength of concrete cubes containing 0% - 25% crushed glass, water to cement ratio ranging from 0.45 – 0.65 and concrete grade M10 – M30 was optimized using the optimization toolbox in MATLAB subject to quadratic programming components. These components are a non-linear objective function, a set of linear equality constraint, a set of linear inequality constraints and set of non-linear constraints. Equation 29 is the objective function of the optimization problem subject to Equation 7 and Equation 30 which are the sets of linear equality constraint and linear inequality bounds. The local maximum of the compressive strength function was found to be 36.83 N/mm^2 derived from a mix ratio of 0.4739: 1: 0.6854: 0.1952: 1.7612 corresponding with water to cement ratio, cement, sand, crushed glass and BNG respectively. The MATLAB code for implementing the optimization is shown in the Appendix.

$$\begin{gathered} 0 \leq X_1 \leq 1 \\ 0 \leq X_2 \leq 1 \\ 0 \leq X_3 \leq 1 \\ 0 \leq X_4 \leq 1 \\ 0 \leq X_5 \leq 1 \end{gathered} \tag{30}$$

3.5.1. MATLAB code for optimization of compressive strength

```
mix_ratio = [0.65 1 2.85 0.15 6; 0.55 1 1.8 0.2 4; 0.6 1 1.28 0.22 3; 0.45 1 0.8 0.2 2; 0.5 1 0.56 0.19 1.5]'
LB = [0;0;0;0;0]
UB = [1;1;1;1;1]
Aeq = [1 1 1 1 1]
beq = 1
x0 = [0;1;0 ;0 ;0]
[solu,val] = fmincon(@objectiveFcn,x0,[],[],Aeq,beq,LB,UB,[]);
val = abs(val)
disp(solu)
disp(val)

max_mix = mix_ratio*solu

function z = objectiveFcn(p_mix)
z = -((16.71*p_mix(1)*(2*p_mix(1)-1))+(16.03*p_mix(2)*(2*p_mix(2)-1))+(23.44*p_mix(3)*(2*p_mix(3)-
1))+(26.67*p_mix(4)*(2*p_mix(4)-1))+(24.67*p_mix(5)*(2*p_mix(5)-
1))+(102.76*p_mix(1)*p_mix(2))+(84.52*p_mix(1)*p_mix(3))+(95.12*p_mix(1)*p_mix(4))+(68.96*p_mix(1)*p_mix
(5))+(85.92*p_mix(2)*p_mix(3))+(95.04*p_mix(2)*p_mix(4))+(91.96*p_mix(2)*p_mix(5))+(76.64*p_mix(3)*p_mix(
4))+(158.52*p_mix(3)*p_mix(5))+(147.24*p_mix(4)*p_mix(5)));
end
```

4. Discussion

4.1. Influence of Mix Gradation on Compressive Strength

Mix gradation plays a crucial role in determining the packing density and mechanical performance of concrete. In this study, the mix proportions were optimized using Scheffe's model, incorporating crushed glass as a partial replacement for fine aggregate. The variations in compressive strength observed in Table 4 indicate that different mix ratios resulted in significant strength differences, with values ranging from 16.71 N/mm² to 39.63 N/mm².

From Table 2 and Table 3, the pseudo and actual mix proportions show different combinations of cement, water, fine aggregate, crushed glass, and Bida Natural Gravel (BNG). The differences in gradation may have influenced the interparticle voids, affecting the overall strength of the concrete. A denser mix with well-graded particles typically exhibits better mechanical interlocking and reduced porosity, leading to higher compressive strength. Samples containing higher percentages of crushed glass (above 20%) exhibited reduced strength in some cases, likely due to the finer particle distribution of crushed glass affecting the void structure. Mixes with balanced ratios of sand and crushed glass showed improved compressive strength due to optimized gradation, leading to better particle packing. The optimal mix ratio identified (0.4739: 1: 0.6854: 0.1952: 1.7612) resulted in a maximum strength of 36.83 N/mm2, indicating that a specific gradation can lead to optimal performance.

4.2. Influence of Glass Particle Distribution on Compressive Strength

The granulometric properties of crushed glass influence its effectiveness as a fine aggregate replacement. The study mentions that crushed glass was sieved to pass a 1.18 mm British Standard sieve, ensuring compatibility with the fine aggregate. However, the distribution of particle sizes within the crushed glass fraction plays a key role in determining its behavior in the mix. If the crushed glass particles are too fine, they may behave similarly to a filler rather than an aggregate, leading to increased demand for cement paste and a potential reduction in workability and strength. The reduced workability noted by Metwally (2007) in similar studies suggests that glass particles influence water demand, which in turn affects compressive strength. The results in Table 4 indicate that mixes with higher water content exhibited lower strength values, further reinforcing this relationship.

Mixes with better-graded aggregates (optimized proportions of sand, BNG, and crushed glass) had higher strengths due to reduced void content. Mixes with a higher w/c ratio tended to have lower strength due to increased porosity, as seen in the lower strength values recorded for some mix designs in Table 4. While a moderate replacement level (10-15%) appeared beneficial, excessive replacement (above 20%) led to a decline in strength, potentially due to reduced bonding and increased brittleness.

5. Conclusions

The 28-day compressive strength of concrete containing crushed glass used as partial replacement for fine aggregate was optimized with the Scheffe's model as the objective function using the MATLAB language. The conclusions of the finding in this research are as follows:

1. It is possible to predict the mix ratios associated with a given compressive strength as well as the compressive strength associated with a given mix ratio using the model equation. The results indicate that crushed glass can partially replace fine aggregate without compromising compressive strength. This offers a sustainable alternative for concrete production, reducing environmental impact while maintaining structural integrity. The methodology is particularly useful for designing eco-friendly concrete mixes tailored to specific strength requirements.
2. The Fischer test ($p > 0.05$) confirmed the model's adequacy, with predictions closely aligning with experimental results ($R^2 = 75.76\%$) as shown in Fig. 4. Using more sample points will significantly improve the accuracy of the model.
3. The Scheffe's model equation reveals the maximum strength of the mix is 36.83 N/mm2 associated with a mix ratio of 0.4739: 1: 0.6854: 0.1952: 1.7612 corresponding to water, cement, sand, crushed glass and BNG to cement ratios respectively. Scheffe's simplex model effectively optimized the 28-day compressive strength of concrete containing crushed glass. The validated model enables accurate prediction of mix ratios for targeted strengths within the studied range. With an optimal compressive strength of 36.83 N/mm², this approach offers a viable path toward sustainable construction materials.

References

[1] Rakshvir, M., & Barai, S. V. **(2006)**. Studies on Recycled Aggregates-Based Concrete, *Waste Management & Research*, 24 (3), 225–233.

[2] Park, S. B., Lee, B. C., & Kim, J. H. **(2004)**. Studies on mechanical properties of concrete containing waste glass aggregate, *Cement and Concrete Research*, 34 (12), 2181–2189.

[3] Shayan, A., & Xu, A. **(2004)**. Value-Added Utilization of Waste Glass in Concrete, *Cement and Concrete Research*, 34 (1), 81–89.

[4] Corinaldesi, V., Gnappi, G., Moriconi, G., & Montenero, A. **(2005)**. Reuse of Ground Waste Glass as Aggregate for Mortars, *Waste Management*, 25 (2), 197–201.

[5] Chen, C. H., Wu, J. K., & Yang, C. C. **(2006)**. Waste E-Glass Particles Used in Cementitious Mixtures, *Cement and Concrete Research,* 36 (3), 449–456.

[6] Metwally, I. M. **(2007)**. Investigations on the Performance of Concrete Made with Blended finely Milled Waste Glass, *Advances in Structural Engineering,* 10 (1), 47–53

[7] Rajasekeran, S. **(2005)**. Optimization mix of High-Performance Concrete by Evolution Strategies Combined with Neural Network, *Indian Journal of Engineering & Material Science*, 13, 7-17.

[8] Chen, H. S., Sun, W., & Stroven, P. **(2003)**. Prediction of compressive strength and optimization of mixture proportioning in ternary cementitious systems, *Eng. Mater. Struct.*, 36(6), 396–401.

[9] Nwakonobi, T. U., & Osadebe, N. N. **(2008)**. Development of an optimization model for mix proportion of clay-rice husk-cement mixture for animal buildings, *Agricultural Eng. Int.: CIGR E-Journal*, Manuscript BC 08 007, Vol. X.

[10] Anyaogu, L. & Ezeh, J. C. **(2013)**. Optimization Compressive strength of fly ash blended Cement Concrete Using Scheffe's Simplex Theory, *Academic Research Natural and Applied Sciences,* (4)2,2223-9553.

[11] Simon, M. **(2003)**. Concrete mixture optimization using statistical method, *Final Report Federal Highway Administration.* Maclean VA, 120-127.

[12] Ezeh, J. C., & Ibearugbulem, O. M. **(2009)**. Application of Scheffe's model in optimization of compressive strength of Rivers Stone Aggregate concrete, *Int. J. Natural and Applied Sciences*, 5(4), 303-308.

[13] Osadebe, N. N. **(2003)**. Generalized mathematical modeling of compressive strength of normal concrete as a multi-variant function of the properties of its constituent components. University of Nigeria Nsukka.

[14] Arimanwa, J. I., Onwuka, D. O., Arimanwa, M. C., & Onwuka, U. S. (2012). Prediction of the Compressive Strength of Aluminum Waste–Cement Concrete Using Scheffe's Theory, Journal of Materials in Civil Engineering, 24, 177-183.

[15] Kalantari, B., Mafian, S., & Huat, B. B. K. **(2009)**. RC Concrete Versus Conventional Concrete Pavement, *Contemporary Engineering Science*, 2(3), 139-148.

[16] Scheffe, H. **(1958)**. Experiments with mixture, Journal of the Royal Statistical Society, 20, 344 – 360.

[17] Simon, M. J., Lagergreen, E. S., & Snyder, K. A. **(1997)**. Concrete Mixture Optimization Using Statistical Mixture Design Methods. Proceedings of the PCI/HWA International Symposium of High-Performance Concrete, New Orlean, 230-244.