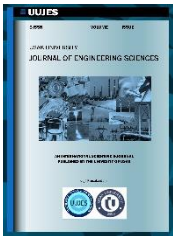

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Research article

A SPIN-1 ISING MODEL INVESTIGATION OF THE MAGNETIC SYSTEM IS CARRIED OUT WITHIN THE CONTEXT OF GENERALIZED STATISTICAL MECHANICS

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Abstract

In this study, magnetization is examined with the help of the Ising model within the framework of non-comprehensive statistical mechanics, where the behavior of the interacting fundamental moment ensemble is taken into account. Researchers employ the spin-1 single lattice Ising model or three-state systems to examine the physical systems with three states and two order parameters. Within this model, various thermodynamic characteristics of phenomena like ferromagnetism in binary alloys, liquid mixtures, liquid-crystal mixtures, freezing, magnetic order, phase transformations, semi-stable and unstable states, ordered and disordered transitions have been investigated for three distinct forms associated with $q < 1$, $q = 1$, and $q > 1$. In this context, q represents the non-extensivity parameter of Tsallis statistics.

Keywords: *Tsallis statistics; Ising model; Magnetization.*

1. Introduction

Understanding the essence of the interaction between magnetic atoms and the magnetic moment value of these atoms holds significant significance when examining ferromagnetism. For this purpose, one of the methods is the study of the magnetic properties of a magnetically dilute system based on the magnetic atoms which are put together with non-magnetic atoms. In 1988, there existed a research on the mathematical properties and physical applications of a new versions of entropy based on generalized entropic functional [1, 2]. Tsallis demonstrated that an entropy functional dependent on q extended the conventional Boltzmann-Gibbs thermostatistical formalism to encompass

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non-extensive systems. Within this approach, q is referred to as the non-extensive parameter. [3-5]. There are important physical scenarios one of which is self-graviting systems that are characterized by a non-extensive behavior. Dilute magnetic systems are examined in this study using non-extensive statistics and spin-1 Ising systems are used as the model's foundation. The general Hamiltonian for spin-1 Ising systems with next nearest pair interactions is

$$H = -J \sum_{\langle ij \rangle} s_i s_j - K \sum_{\langle ij \rangle} s_i^2 \, s_j^2 - D \sum_{j=1}^{N} s_i^2 - L \sum_{\langle ij \rangle} \left(s_i^2 s_j + s_i s_j^2 \right) - \mathrm{H} \sum_i s_i \tag{1}$$

where, J, K, D, L, and H are constants for bilinear exchange interaction, bi-quadratic exchange interaction, crystal field interaction, magnetic field due to s, and dipole-quadrupole interaction, respectively. The term "H" refers to the magnetic perturbation of the third degree, while "L" represents the number of lattice points. When the systems exist in semi-stable states or phases, their properties undergo significant alterations. [6-9]. All terms that are conceivable when $s_i^j = s_i$ are included in the Hamiltonian generated by Eq. (1), but higher order powers of spin are excluded.

2. Materials and Method

A dilute magnetic system is one that is created when magnetic and nonmagnetic atoms combine. In the case of such systems, the potential for displaying magnetic characteristics arises when the concentration of magnetic atoms attains a specific threshold [10,11]. The Hamiltonian of such a system is expressed in the form

$$H = -\sum_{\langle ij \rangle} s_i s_j J n_i n_j \tag{2}$$

This is determined from Eq. (1) by adopting K=D=L=H=0. In this equation, J is the energy resulting from the exchange interaction, and s_i and j_j are the spin vectors of the i^{th} and j^{th} atoms, respectively. n_i and n_j represent spin disorder variables, which can take on the values of 0 and 1, respectively. The mean value of these variables determines the magnetic concentration. [10,11].

2.1 The System's Free Energy

This system's energy per atom is:

$$\frac{E}{N} = J \rho \sigma_{ij} \beta \mathrm{B} \tag{3}$$

In this context, ρ denotes the overall count of lattice points within the system, Λ signifies the number of nearest neighboring lattice points, represents the external magnetic field, and $\beta = \frac{1}{k_\beta T}$. The statistical weight of the spin-1 Ising system can be represented as follows: utilizing the internal variable x_i, which has been defined for three-state systems, along with the double variable σ_{ij} [12].

$$[W]^{\frac{1}{N}} = \frac{\left[\prod_{i=1}^{7} (p_i Y)!\right]^{\rho-1}}{L!^{\frac{\rho}{2}-1} \left[\prod_{i,j=1}^{3} \left(\sigma_{ij} L\right)!\right]^{\frac{\rho}{2}}} \tag{4}$$

Here, Y represents the count of systems in the ensemble, while N denotes the total number of lattice points within the system. On the other hand, everyone is familiar with what entropy is:

$$S = k_B \ln W \quad . \tag{5}$$

When one system's entropy is calculated (Y=1).

$$\frac{S}{N} = k_B[(\rho - 1)\sum_{i=1}^{7} x_i ln x_i - (\frac{\rho}{2})\sum_{i,j=1}^{7} \sigma_{ij} ln\sigma_{ij}]. \quad (6)$$

2.2 The State of Equilibrium for the Dilute System within the Context of Non-extensive Statistical Mechanics

Prior to 1998, all physical properties of statistical systems were examined using Boltzmann-Gibbs statistics. Based on Boltzmann-Gibbs statistics, macroscopic quantities like free energy, entropy, and internal energy of a statistical system are perceived as extensive variables. In 1998, a thermodynamically motivated generalization was conducted to comprehend the structure or resolve numerous unfamiliar systems. This generalization drew inspiration from the probability definition of multifractal geometry. Magnetization is a phenomenon characterized by long-range interactions and memory. Systems that exhibit these characteristics are referred to as non-extensive systems. Hence, the investigation of magnetization is conducted within the framework of non-extensive statistical mechanics. This generalization involves parameterizing all statistical quantities with a parameter q. In the limit as q approaches 1, the statistics being studied converges to the standard Boltzmann-Gibbs statistics. However, for values of q other than 1, macroscopic quantities such as internal energy, free energy, and entropy in Boltzmann-Gibbs statistics are not considered extensive quantities; in other words, they are non-extensive [13]. Mathematical terms from non-extensive statistical mechanics are introduced with the aim of generalization;

$$ln_q p = \frac{p^{1-q}}{1-q} \quad (7)$$

3. Results and discussion

3.1 Determination of the Physical Quantities

The temperature dependence of the generalized magnetization, generalized magnetic susceptibility and generalized specific heat are greatly governed by the values assigned to the nonextensivity parameter q [14, 15].
The following is the result of determining the relationship between the parameter t and temperature using probability, nonlinear equations, and expressions linked to concentration:

$$e_q^{\frac{J}{kT}} = \frac{n_\alpha tanh_q \rho t}{cosh_q 2(\rho-1)t[n_{\alpha\alpha} tanh_q 2(\rho-1)t + n_{\alpha\beta} tanh_q(\rho-1)t - n_\alpha tanh_q \rho t]} - \frac{n_{\alpha\beta} tanh_q(\rho-1)t}{cosh_q 2(\rho-1)t[n_{\alpha\alpha} tanh_q 2(\rho-1)t + n_{\alpha\beta} tanh_q(\rho-1)t - n_\alpha tanh_q \rho t]} \quad (8)$$

The Fig. 1 illustrates the plot of the function M_q=f(T), showcasing magnetization vary with temperature for a specific concentration value (c=1) across different q values. In the figure the yellow, orange, gray and the blue lines refer to q = 1.2, q = 1.0, q = 0.7 and q = 0.4 respectively.

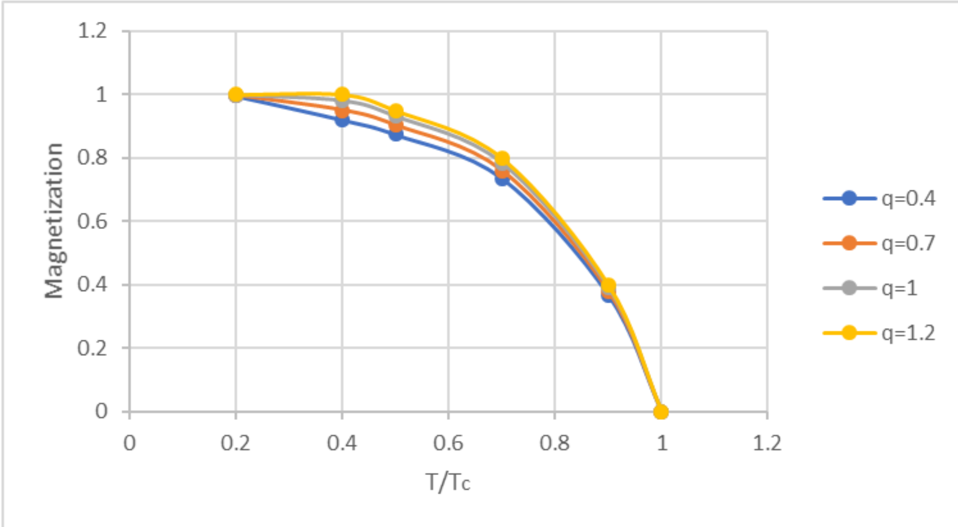

Fig. 1 The plot represents the magnetization vary with temperature for a simple cubic structure at a specific concentration, considering four different q values.

For q < 1 the paramagnetic ferromagnetic phase transition is second order. On the other hand, the expression for the susceptibility is as follows.

$$\frac{1}{\chi_q} = \frac{kT\left[1+e_q^{\frac{J}{kT}}cosh_q 2(\rho-1)t\right](n_\alpha-\rho n_{\alpha\alpha}-n_{\alpha\alpha})+2n_{\alpha\alpha(\rho-1)}}{G^2\left[1+e_q^{\frac{J}{kT}}cosh_q 2(\rho-1)t\right](n_\alpha^2+n_{\alpha\alpha}n_\alpha)-n_\alpha n_{\alpha\alpha}} \tag{9}$$

Fig. 2 illustrates the plot of the function χ_q=f(T), presenting the susceptibility vary with temperature at a particular concentration value (c=1) for two distinct q values. In this plot yellow line corresponds to q = 1.2 and the blue line corresponds to q=0.4.
The Fig. 1 illustrates the plot of the function M_q=f(T), showcasing magnetization vary with temperature for a specific concentration value (c=1) across different q values. In the figure the yellow, orange, gray and the blue lines refer to q = 1.2, q = 1.0, q = 0.7 and q = 0.4, respectively.

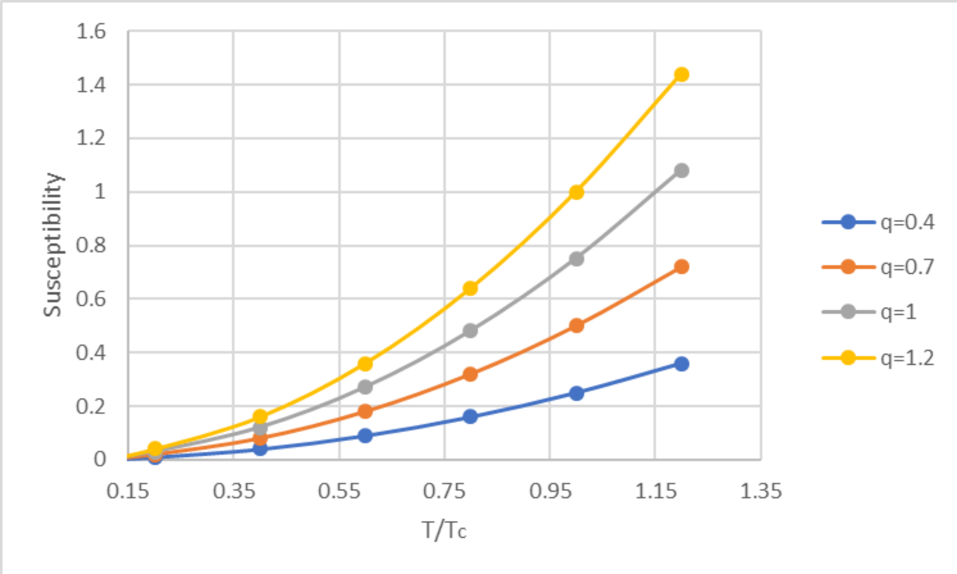

Fig. 2 Shows how a basic cubic structure's sensitivity to heat varies with respect to temperature for a given concentration and for four distinct q values.

When the energy's magnetic component is taken into account, one writes:

$$\frac{E_q}{N} = \frac{\rho J}{4}\left(4\eta - n_{ij}\right) \tag{10}$$

$$\frac{E_q}{N} = \frac{-\rho^J}{4} n_{\alpha\alpha}\left[\frac{1-e^{\frac{-J}{kT}}\sec(h_q)2(\rho-1)t}{1+e^{\frac{-J}{kT}}\sec(h_q)2(\rho-1)t}\right] \tag{11}$$

Using these expressions, the specific heat is found to be;

$$\frac{C_q}{kN} = \frac{\rho^J}{4k} n_{\alpha\alpha}\frac{d}{dT}\left[\frac{e^{\frac{-J}{kT}}\sec(h_q)2(\rho-1)t-1}{e^{\frac{-J}{kT}}\sec(h_q)2(\rho-1)t-1}\right] \tag{12}$$

In Fig. 3, the plot of C_q=f(T), showcasing the fluctuation of specific heat with respect to temperature for a specific concentration value (c=1) across three different q values, is displayed. The lines in yellow, orange, gray, and blue represent q values of 1.2, 0.7 and 0.4, respectively.

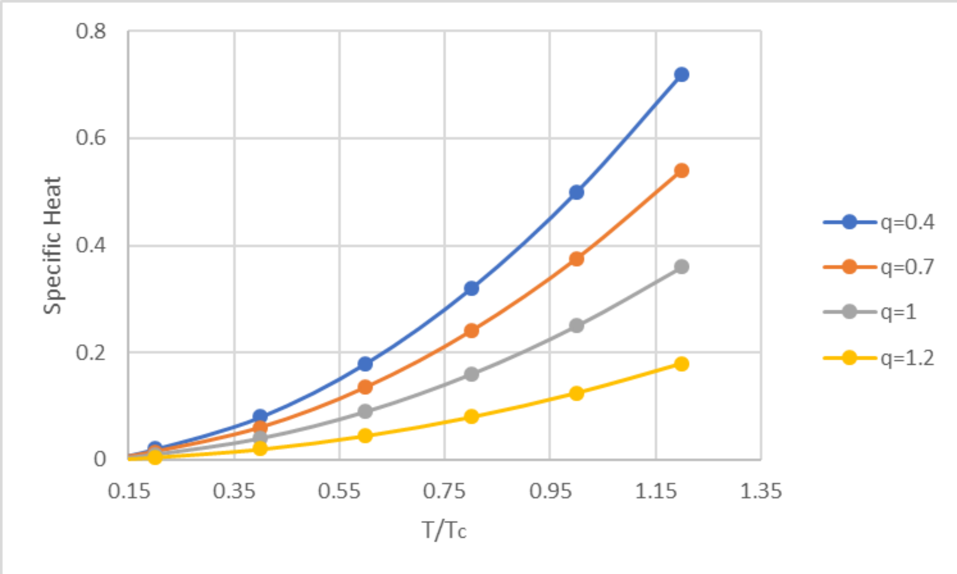

Fig. 3 Illustrates the relationship between temperature and specific heat for a simple cubic structure at a specific concentration, considering four distinct values of q.

4. Conclusion

In this study, the spin-1 Ising model is used to analyze the microscopic properties of magnetic systems viewed as a collection of interacting fundamental moments. Using statistical mechanics, a bridge has been established between the microscopic approach and macroscopic experimental data. The long-range phenomenon of magnetization also has a memory effect. These systems have the attribute of non-extensivity. Because of this, magnetization has been taken into account within the context of non-extensive statistical mechanics. The generalization process begins with the conventional method. There are also several experimental investigations [14, 15] that look into this sort of system. This study investigates the temperature-dependent fluctuations in magnetization, susceptibility, and specific heat. It has been shown that for the entropic index q, as q decreases, the magnetization shows a linear fluctuation rather than a parabolic fluctuation. However, the susceptibility does not alter with the q values and the specific heat, temperature dependency rises as q decreases.

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