

Implementation of Fuzzy Linear Programming Approach for More Accurate Demand Forecasting in a Make-to-Stock Company

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Abstract

Demand forecasting is of crucial importance for make-to-stock companies because product demand is uncertain and it changes with time. Fuzzy linear programming (FLP) can be an optimum approach for such uncertain situations. In this study, the FLP model was used for more accurate demand forecasting in a make-to-stock company. Demand forecasting study was carried out according to the FLP method and linear programming (LP) method. Solutions of FLP and LP were compared in terms of imputed shortage cost, inventory carrying cost, and net profit. Results show that the applied FLP method is more advantageous than LP as it provides a 67% decrease in costs and a 15% increase in net profit.

Keywords: Demand Forecasting; Fuzzy Linear Programming; Make-to-Stock.

1. INTRODUCTION

Determination of product demand is very important in terms of production planning since companies make the production plan according to product demand. On the other hand, demand amounts in make-to-stock companies can't be known precisely. Therefore, they make the production plan in accordance with the estimated demand value.

Make-to-stock companies aim to forecast the product demand with a minimum error as more realistic demand forecasting enables to decrease in the inventory carrying cost and imputed shortage cost. In addition, when companies achieve this aim, costs based on the deterioration of product inventory decline. However, improving demand forecast accuracy is a difficult issue because product demand is unknown and it varies from period to period. Thus, fuzzy logic is a suitable approach to deal with decision problems involving such uncertainties and linear programming (LP) [1]. Zimmermann introduced a fuzzy approach regarding LP problems in 1974. In addition, Lai and Hwang, Tong Shaocheng, Buckley, among others, cope with the fact that all parameters are fuzzy [2].

When evaluated similar studies, Azizi et al. [3] developed adaptive neuro fuzzy inference system to predict the throughput of production systems. It was used because of factors include uncertainty such as set up time and demand. It was applied in tile industry. Obtained results revealed that adaptive neuro fuzzy inference system is appropriate for prediction of throughput in uncertainty. Worapradya and

Thanakijkasem [4] developed a model with hierarchical genetic algorithm to optimize the steel production. Figueroa-Garcia et al. [5] used fuzzy linear programming (FLP) and interval fuzzy set approach for mixed production planning problem. Torabi et al. [6] used fuzzy hierarchical production planning approach for solving multi-level production planning problem. Bilgen [7] utilized fuzzy mathematical programming approach for production and distribution planning problem. Ertuğrul and Tuş [8] investigated the interactive FLP approach by utilizing Zimmermann, Werners, Chanas and Verdegay approaches. Their application was performed in the textile industry.

In recent years, FLP has been used in areas such as finance, energy, and transportation [9-15]. In the literature, no study has been found in which FLP has been compared with classical LP in the demand forecasting problem of a make-to-stock company. In this respect, it is expected that this study will make a significant contribution to the literature.

In this paper, production planning problem in a make-to-stock company was investigated and a demand forecasting study was conducted. FLP approach was used to forecast the product demand accurately. Because of the fact that both objective function and constraints include uncertainty, Zimmermann's approach was used. Estimated product demands were obtained from the models of classical LP and FLP. Production planning was performed for each model by using these demand values for three periods. Results were compared with the aspect of inventory carrying cost, imputed shortage cost, and net profit.

2. FUZZY SETS

In classical sets, there are two states, 0 and 1. In other words, an element either belongs to a set or doesn't [16]. For instance, it is supposed that there is a car. The headlights, air conditioner and windscreen wiper of the car are out of service. However, the engine of the car is running and it is moving. In that case, it can't be reached a conclusion that either the car is out of order exactly or it runs precisely. Therefore, it is more suitable to characterize the car as a slight level failed and middle level failed. As is also understood from this example, an element can belong to a set with any value between 0 and 1. This value is entitled as the degree of membership and symbolized with "μ" for fuzzy sets fundamental to fuzzy logic. Also, degrees of membership are expressed with membership functions.

It can be propounded that there are three basic reasons for using fuzzy logic. The first of these is subjectivity. Judgements related to a situation can vary from person to person. For example, we assume there are three age sets; young, middle and old. While thirty age belongs to 'young' set for a person, it belongs to 'middle' set for another person. Secondly, whole elements of a set can't belong to it with the same degree of membership. In addition, an element can belong to sets more than one [17]. Thirdly, real world problems include many variables. Each variable's effect to problem and interactions among them can't be determined exactly. In other words, this is very difficult to model this type of problems. Thus, fuzzy logic can be an ideal approach for solving them.

For instance, it is presumed that there are five sets about temperature; very low, low, middle, high and very high. In this case, we have five fuzzy sets and five membership functions. These are illustrated below:

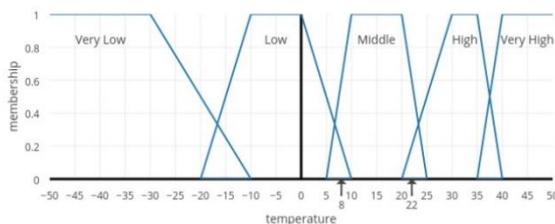


Figure 1. Fuzzy sets and membership functions

As shown in Figure 1, each element of a set can't belong to it with the same degree of membership. For example, whereas 15C° belongs to 'middle' set with 1 degree of membership, 8C° belongs to it with 0.4 degree. Also, 22C° belongs to 'high' set with 0.2 degree of membership and 'middle' set with 0.6 degree. In other words, it belongs to two sets.

In fuzzy logic, triangular and trapezoidal membership functions are used generally. Additionally, membership functions such as gaussian and sigmoid are also available. While determining to suitable membership function for a problem, capability of representation the data in it of membership function must be taken into account [18].

3. FUZZY LINEAR PROGRAMMING

3.1. Linear Programming (LP)

LP is a type of mathematical programming. The aim of mathematical programming is to determine the extremum point of a function under some constraints. Both objective function and constraints are linear in LP [19]. LP model can be loosely formulated as follows:

$$\begin{aligned} &Max C^T x \\ &subject\ to \\ &Ax \leq b \\ &x \geq 0 \end{aligned} \tag{1}$$

in which x represents the vector of variables, C is the vector of coefficients in objective function, (.)^T is the matrix transpose, A is a matrix of coefficients in constraints, b is a vector of right-hand sides of the constraints. The expression to be max or min is called objective function.

LP is of crucial importance and it is used in many fields such as engineering and economy [20]. LP problems can be solved with LINDO optimization software. If the objective function, right-hand sides or coefficients of constraints are uncertain, LP isn't appropriate to solve the problem.

Many types of FLP were formed. Some methods were developed for converting them to LP. Finally, they were solved with optimization software [21].

3.2. Fuzzy Linear Programming (FLP)

Sometimes, coefficient of the variable belongs to a problem depends on other parameters. Also, it can't be evaluated exactly and can determine subjectively. Thus, coefficients in problem can include uncertainty. In this case, classical LP falls behind with the solution of the problem. FLP can be an ideal approach for such problems [22]. Formulation of the FLP model can be considered as follows:

$$\begin{aligned} &Max \tilde{Z} = \tilde{C}^T x \\ &subject\ to \\ &\tilde{A}x \leq \tilde{b} \\ &x \geq 0 \end{aligned} \tag{2}$$

in which tilde (~) is fuzzy symbol differently from LP model. To solve this problem, it is necessary to find the probability distribution of the optional objective function Z. To deal with this problem, fuzzy constraints and fuzzy objective function convert into crisp ones [23].

3.3. Zimmermann Method

This method is one of the FLP models. Both objective function and constraints are fuzzy in this method. An FLP model developed by Zimmermann is indicated as follows [21]:

$$\begin{aligned}
 &C^T x \geq b_0 \\
 &(Ax)_i \leq b_i \\
 &x \geq 0 \quad i = 1, 2, \dots, m
 \end{aligned} \tag{3}$$

A symmetrical model in which the objective function becomes a constraint is the inequality. To arrive at a general formulation, this inequality is converted into matrix form as follows:

$$-C^T x \leq -b_0 \tag{4}$$

in which

$$B = \begin{bmatrix} -C \\ A_i \end{bmatrix} \quad b = \begin{bmatrix} -b_0 \\ b_i \end{bmatrix} \tag{5}$$

The constraint's inequalities mean "as small as or equal to" can be allowed to violate right-hand side b by expanding a value. The degree of violation is represented by membership function as:

$$\mu_0(x) = \begin{cases} 0 & ,if\ Cx \leq b_0 - d_0 \\ 1 - \frac{b_0 - Cx}{d_0} & ,if\ b_0 - d_0 \leq Cx \leq b_0 \\ 1 & ,if\ Cx \leq b_0 \end{cases} \tag{6}$$

$$\mu_i(x) = \begin{cases} 0 & ,if\ (Ax)_i \geq b_i + d_i \\ 1 - \frac{(Ax)_i - b_i}{d_i} & ,if\ b_i \leq (Ax)_i \leq b_i + d_i \\ 1 & ,if\ (Ax)_i \leq b_i \end{cases} \tag{7}$$

in which d is a matrix of admissible violation. This problem can be expressed as follows using an auxiliary variable represented by λ:

$$\begin{aligned}
 &\mu_0(x) \geq \lambda \\
 &\mu_i(x) \geq \lambda \\
 &\lambda \in [0,1]
 \end{aligned} \tag{8}$$

Then, the problem can be stated as LP as follows:

$$\begin{aligned}
 &Max\ \lambda \\
 &s.t. \\
 &\mu_0(x) \geq \lambda \\
 &\mu_i(x) \geq \lambda \\
 &\lambda \in [0,1]
 \end{aligned} \tag{9}$$

This problem is illustrated by the membership functions of the fuzzy objective function and fuzzy constraints as follows:

$$\begin{aligned}
 &Max\ \lambda \\
 &s.t. \\
 &1 - \frac{b_0 - Cx}{d_0} \geq \lambda \\
 &1 - \frac{(Ax)_i - b_i}{d_i} \geq \lambda, \forall i \\
 &\lambda \in [0,1] \\
 &x \geq 0
 \end{aligned} \tag{10}$$

The final form of the FLP model is obtained after some simplifications as follows:

$$\begin{aligned}
 &Max\ \lambda \\
 &s.t. \\
 &C^T x - \lambda d_0 \geq b_0 - d_0 \\
 &(Ax)_i + \lambda d_i \leq b_i + d_i, \forall i \\
 &\lambda \in [0,1] \\
 &x \geq 0
 \end{aligned} \tag{11}$$

4. IMPLEMENTATION

Demand forecasting study was conducted with data obtained from a make-to-stock company. The problem was built as FLP model because demand of the product types and expected profit are uncertain. Solution of this model gave estimated demand values for each product type per month to maximize the total profit. Since the company follows make-to-stock strategy, estimated demand values are equal to production amounts. Production data and constraints are given in Table 1.

4.1. Classical LP Model and Solution

Problem was modelled as monthly basis by using related data in Table 1. The classical LP model of the problem is given Eq. (12).

Table 1. Application data

Variables for products	X ₁	X ₂	X ₃	X ₄	X ₅	X ₆	X ₇	X ₈	X ₉
Profits (TRY per tonne)	1509	1509	503	1509	1006	503	1509	1006	503
Expected demands (tonne per month)	25	100	150	30	150	200	30	250	80
Tolerances for demands (tonne per month)	5	50	40	5	40	50	5	40	20
Labor usage (hour per tonne)	0.333	0.333	0.333	0.283	0.283	0.283	0.3	0.3	0.3
Expected profit (TRY)	1,200,000								
Tolerance for profit (TRY)	200,000								
Monthly production capacity (tonne)	1150								
Monthly labor capacity (hour)	364								

$$\begin{aligned}
 &Max\ 1509x_1 + 1509x_2 + 503x_3 + 1509x_4 + 1006x_5 + 503x_6 + 1509x_7 + 1006x_8 + 503x_9 \\
 &subject\ to \\
 &x_1 \geq 25 \\
 &x_2 \geq 100 \\
 &x_3 \geq 150 \\
 &x_4 \geq 30 \\
 &x_5 \geq 150 \\
 &x_6 \geq 200 \\
 &x_7 \geq 30 \\
 &x_8 \geq 250 \\
 &x_9 \geq 80 \\
 &0.333x_1 + 0.333x_2 + 0.333x_3 + 0.283x_4 + 0.283x_5 + 0.283x_6 + 0.3x_7 + 0.3x_8 + 0.3x_9 \leq 364 \\
 &x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 \leq 1150 \\
 &x_i \geq 0 \\
 &i = 1,2,\dots,9
 \end{aligned}
 \tag{12}$$

$$\begin{aligned}
 &Max\ \lambda \\
 &subject\ to \\
 &1509x_1 + 1509x_2 + 503x_3 + 1509x_4 + 1006x_5 + 503x_6 + 1509x_7 + 1006x_8 + 503x_9 - 200,000\lambda \geq 1,000,000 \\
 &x_1 + 5\lambda \leq 30 \\
 &x_2 + 50\lambda \leq 150 \\
 &x_3 + 40\lambda \leq 190 \\
 &x_4 + 5\lambda \leq 35 \\
 &x_5 + 40\lambda \leq 190 \\
 &x_6 + 50\lambda \leq 250 \\
 &x_7 + 5\lambda \leq 35 \\
 &x_8 + 40\lambda \leq 290 \\
 &x_9 + 20\lambda \leq 100 \\
 &0.333x_1 + 0.333x_2 + 0.333x_3 + 0.283x_4 + 0.283x_5 + 0.283x_6 + 0.3x_7 + 0.3x_8 + 0.3x_9 \leq 364 \\
 &x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 + x_9 \leq 1150 \\
 &\lambda \in [0,1] \\
 &x_i \geq 0 \quad i = 1,2,\dots,9
 \end{aligned}
 \tag{13}$$

LINDO optimization software was used to solve the LP model of the problem. The relevant solution is given in Table 2.

Table 2. Results of LP model

Variable	Value	Reduced Cost
Objective Function	1,101,570 TRY	-
X ₁	160	0
X ₂	100	0
X ₃	150	0
X ₄	30	0
X ₅	150	0
X ₆	200	0
X ₇	30	0
X ₈	250	0
X ₉	80	0

4.2. FLP Model and Solution

Problem was modelled as monthly basis by using related data in Table 1. The FLP model of the problem is given Eq. (13).

LINDO software was used to solve the FLP model of the problem. The relevant solution is given in Table 3.

Table 3. Results of FLP model

Variable	Value	Reduced Cost
λ	0.234	0
X ₁	28.832	0
X ₂	138.321	0
X ₃	180.657	0
X ₄	33.832	0
X ₅	180.657	0
X ₆	177.885	0
X ₇	33.832	0
X ₈	280.657	0
X ₉	95.329	0

4.3. Comparison Between FLP and LP

Quarterly production plans of each model were made by using estimated demand values obtained from LP and FLP.

Outputs of production plans were compared in terms of inventory carrying cost, imputed shortage cost and net profit.

The company can carry inventory maximum 3 months because of the deterioration problem of products. Monthly inventory carrying cost for the company is 12% of product profit per tonne. The company practices back order strategy.

If it meets back orders next month, imputed shortage cost for stock-out products is equal to 50% of unit profit. If it meets after two months, the cost is equal to 75% of unit profit. Otherwise, the cost is equal to unit profit.

Production plans of LP and FLP are shown in Table 4 and Table 5.

Table 4. Production planning of LP model

	P (t-2)	D (t-2)	IS (t-2)	P (t-1)	D (t-1)	IS (t-1)	P (t)	D (t)	IS (t)	Descriptions
X ₁	160	30	130 (I)	160	27	263 (I)	160	29	394 (I)	t: Time period (month), P: Production (tonne), D: Demand (tonne), IS: Inventory Status, I: Inventory, S: Stock-out 20₀: 20 items can't be met in this period, they can be met next period.
X ₂	100	120	20 ₀ (S)	100	113	20 ₁ +13 ₀ (S)	100	138	20 ₂ +13 ₁ +38 ₀ (S)	
X ₃	150	147	3 (I)	150	149	4 (I)	150	162	8 ₀ (S)	
X ₄	30	28	2 (I)	30	33	1 ₀ (S)	30	31	-	
X ₅	150	160	10 ₀ (S)	150	152	10 ₁ +2 ₀ (S)	150	157	10 ₂ +2 ₁ +7 ₀ (S)	20₂+13₁+38₀: 20 items are stock-out since two period, 13 items are stock-out since one period and 38 items are stock-out in last period.
X ₆	200	190	10 (I)	200	195	15 (I)	200	184	31 (I)	Profit Status
X ₇	30	31	1 ₀ (S)	30	32	1 ₁ +2 ₀ (S)	30	34	1 ₂ +2 ₁ +4 ₀ (S)	Total Profit = $\sum_{x=1}^9$ Estimated production amount x Unit profit
X ₈	250	257	7 ₀ (S)	250	261	7 ₁ +11 ₀ (S)	250	260	7 ₂ +11 ₁ +10 ₀ (S)	Net Profit = Total Profit – (Inventory carrying cost + Imputed shortage cost)
X ₉	80	83	3 ₀ (S)	80	84	3 ₁ +4 ₀ (S)	80	89	3 ₂ +4 ₁ +9 ₀ (S)	Net Profit = 1,101,570 - (146,673.84 + 123,863.75) = 831,032.4 TRY

Table 5. Production planning of FLP model

	P (t-2)	D (t-2)	IS (t-2)	P (t-1)	D (t-1)	IS (t-1)	P (t)	D (t)	IS (t)	Net Profit
X ₁	28.832	30	1.168 ₀ (S)	28.832	27	0.664 (I)	28.832	29	0.496 (I)	958,115.4 TRY
X ₂	138.321	120	18.321 (I)	138.321	113	43.642 (I)	138.321	138	43.963 (I)	
X ₃	180.657	147	33.657 (I)	180.657	149	65.314 (I)	180.657	162	83.971 (I)	
X ₄	33.832	28	5.832 (I)	33.832	33	6.664 (I)	33.832	31	9.496 (I)	
X ₅	180.657	160	20.657 (I)	180.657	152	49.314 (I)	180.657	157	72.971 (I)	
X ₆	177.885	190	12.115 ₀ (S)	177.885	195	12.115 ₁ +17.115 ₀ (S)	177.885	184	12.115 ₂ +17.115 ₁ +6.115 ₀ (S)	
X ₇	33.832	31	2.832 (I)	33.832	32	4.664 (I)	33.832	34	4.496 (I)	
X ₈	280.657	257	23.657 (I)	280.657	261	43.314 (I)	280.657	260	63.971 (I)	
X ₉	95.329	83	12.329 (I)	95.329	84	23.658 (I)	95.329	89	29.987 (I)	

5. CONCLUSION

In this paper, demand forecasting problem of a make-to-stock company was addressed. Because the company follows make-to-stock strategy, it needs estimated product demand values to develop a production planning. However, product demand cannot be known precisely and it varies from period to period. Because of this uncertainty, fuzzy logic can be an ideal approach for estimating the product demand with a minimum error.

As estimation of product demand was more accurate, Zimmerman's FLP approach was used. Amount of production for each product type was obtained from the solutions of FLP and LP. Output of production plans based

on solutions of FLP and LP models were compared in terms of costs and net profit. Results showed that when fuzzy logic and LP was used together, product demands were forecasted more accurately. Therefore, inventory carrying cost and imputed shortage cost reduced by 67% totally and also net profit increased by 15%.

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