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Traveling Wave Solutions of Some Nonlinear Partial Equations

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Abstract

We apply the extended trial equation method (ETEM) to obtain exact solutions of (2+1) dimensional nonlinear electrical transmission line equation (NETLE) and Benjamin-Bona-Mahony-Peregrine (BBMP) equation in this study. We create some exact solutions like soliton solutions, rational, Jacobi elliptic, periodic wave solutions and hyperbolic function solutions of these equations via ETEM. After that, we present conclusions that we acquired thanks to this method.

Keywords: *(2+1) dimensional nonlinear electrical transmission line equation, Benjamin-Bona-Mahony-Peregrine equation, extended trial equation method, rational, Jacobi elliptic and hyperbolic function solutions.*

2010 Mathematics Subject Classification: *35C07, 35A20, 35A25.*

1. Introduction

Recently, travelling wave solutions are a very significant subject in biophysics, chemistry, optical fibers, technology of space, electricity and different subject in nonlinear sciences. According to recent studies, various researchers have submitted various methods to get travelling wave solutions of NLEEs such as Hirota's direct method [1], Jacobi elliptic function method [2], new version of the trial equation method [3], (G'/G)–expansion method [4], tanh-coth method [5] and many more[6, 7, 8, 9]. In this work, the ETEM [10, 11, 12, 13, 14] will be implemented to obtain exact solutions of (2+1) dimensional NETLE and BBMP equations.

We handle the following (2+1) dimensional NETLE [15]:

$$\frac{\partial^2}{\partial t^2}\left(U-\alpha U^2+\beta U^3\right)-u_0^2\left(\delta_1^2\frac{\partial^2 U}{\partial x^2}+\frac{\delta_1^4\partial^4 U}{12\partial x^4}\right)-\omega_0^2\left(\delta_2^2\frac{\partial^2 U}{\partial y^2}+\frac{\delta_2^4\partial^4 U}{12\partial y^4}\right)=0, \tag{1.1}$$

where α, β, u_0 and ω_0 are constants. δ_1 and δ_2 are the space between two adjoining sections in the longitudinal direction and transverse direction respectively. Md. Abdul Kayum et al. have found soliton solutions of this equation by using modified simple equation method [16]. E. Tala-Tebue et al. have acquired exact soliton solutions of Eq. (1.1) via (G'/G)-expansion method and new Jacobi elliptic function expansion method [17, 18]. M. T. Gulluoglu has found travelling wave solutions of Eq. (1.1) by developed Bernoulli sub-equation function method [19].

Secondly, we investigate the following BBMP equation [20]:

$$\frac{\partial u}{\partial t}+a\frac{\partial u}{\partial x}+bu^q\frac{\partial u}{\partial x}+c\frac{\partial^3 u}{\partial^2 x\partial t}=0, \tag{1.2}$$

where a, b, c and q are nonzero steadies. q power symbolizes the power-law nonlinearity parameter and it is essential to have $q\neq 0$, because these values will location Eq. (1.2) beyond the linear regime. In Eq. (1.2) the initial term specifies evolution term, while the latest term specifies the dispersion term. The third term is the nonlinear term. Khalique has found exact wave solutions of Eq. (1.2) using Lie symmetry method and simplest equation approach [21]. Aminikhah et al. have used the functional variable method to solve this equation [22]. The article is regulated as: In chapter 2, ETEM has been applied to (2+1) dimensional NETLE and BBMP equations. In chapter 3, The obtained results by this method are presented.

2. Fundamentals of the ETEM

Step1: On account of a known nonlinear partial differential equation

$$P(u,u_t,u_x,u_{xx},\ldots)=0, \tag{2.1}$$

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get the wave transformation

$$u(x_1,x_2,...,x_N,t)=u(\eta),\eta=\lambda\left(\sum_{j=1}^{N}x_j-ct\right), \tag{2.2}$$

where $\lambda\neq 0$ and $c\neq 0$. Embedding Eq. (2.2) into Eq. (2.1) satisfies a nonlinear ordinary differential equation,

$$N(u,u^{'},u^{''},\cdots)=0. \tag{2.3}$$

Step2: Take conversion and trial equation as follows:

$$u=\sum_{i=0}^{\delta}\tau_i\Gamma^i, \tag{2.4}$$

where

$$\left(\Gamma'\right)^2=\Lambda(\Gamma)=\frac{\phi(\Gamma)}{\psi(\Gamma)}=\frac{\xi_\theta\Gamma^\theta+...+\xi_1\Gamma+\xi_0}{\zeta_\varepsilon\Gamma^\varepsilon+...+\zeta_1\Gamma+\zeta_0} \tag{2.5}$$

Taking into account correlates Eqs. (2.4) and (2.5), we can have

$$\left(u'\right)^2=\frac{\phi(\Gamma)}{\psi(\Gamma)}\left(\sum_{i=0}^{\delta}i\tau_i\Gamma^{i-1}\right)^2, \tag{2.6}$$

$$\left(u''\right)=\frac{\phi'(\Gamma)\psi(\Gamma)-\phi(\Gamma)\psi'(\Gamma)}{2\psi^2(\Gamma)}\left(\sum_{i=0}^{\delta}i\tau_i\Gamma^{i-1}\right)+\frac{\phi(\Gamma)}{\psi(\Gamma)}\left(\sum_{i=0}^{\delta}i(i-1)\tau_i\Gamma^{i-2}\right), \tag{2.7}$$

where ϕ and ψ are polynomials. Placing these terms into Eq. (2.3) ensures an equation of polynomial $\Omega(\Gamma)$ of Γ:

$$\Omega(\Gamma)=\sigma_s\Gamma^s+...+\sigma_1\Gamma+\sigma_0=0. \tag{2.8}$$

In accordance with balance principle, the correlation of θ, ε and δ can be described and thus values of θ, ε and δ can be received.
Step 3. If all of $\Omega(\Gamma)$ coefficients are zero, a system of algebraic equations is obtained:

$$\sigma_i=0,i=0,...,s. \tag{2.9}$$

Solving equation system (2.9), the values of $\xi_0,...,\xi_\theta;\zeta_0,...,\zeta_\varepsilon$ and $\tau_0,...,\tau_\delta$ can be described
Step4: Simplify Eq. (2.5) to basic integral form,

$$\pm(\eta-\eta_0)=\int\frac{d\Gamma}{\sqrt{\Lambda(\Gamma)}}=\int\sqrt{\frac{\psi(\Gamma)}{\phi(\Gamma)}}d\Gamma. \tag{2.10}$$

Performing a complete distinction system for polynomial to categorize the roots of $\phi(\Gamma)$, we resolve the infinite integral (2.10) and classify exact solutions to Eq. (2.1) by Mathematica.

3. Implementations of the ETEM

In this chapter, we implement the method explained in Chapter 2 to the (2+1)-dimensional NETLE and the BBMP equations, respectively.

3.1. Implementation of the (2+1)-dimensional NETLE

In an attempt to find travelling wave solutions of Eq. (1.1), we take the transformation

$$U(x,y,t)=U(\eta),\eta=\sqrt{k}(x+y-v_0t). \tag{3.1}$$

where v_0 is an arbitrary constant and the speed of the travelling wave. Then, we attain

$$\left[m-\left(u_0^2K_1+\omega_0^2K_2\right)\right]U+m\left(\beta U^3-\alpha U^2\right)-\frac{1}{12}\left(u_0^2K_1^2+\omega_0^2K_2^2\right)\frac{d^2U}{d\eta^2}=0, \tag{3.2}$$

where $m=kv_0^2,K_1=\delta_1^2k,K_2=\delta_2^2k$. Embedding Eqs. (2.6) and (2.7) into Eq. (3.2), and making use of the balance principle, we gain

$$\vartheta=2\delta+\varepsilon+2. \tag{3.3}$$

Then, we procure the corollaries as:
Case 1: If we select $\varepsilon=0,\delta=1$ and $\vartheta=4$, we have

$$(u')^2=\frac{\tau_1^2\left(\xi_0+\Gamma\xi_1+\Gamma^2\xi_2+\Gamma^3\xi_3+\Gamma^4\xi_4\right)}{\zeta_0} \tag{3.4}$$

$$u'' = \frac{\tau_1\left(\xi_0 + 2\Gamma\xi_2 + 3\Gamma^2\xi_3 + 4\Gamma^3\xi_4\right)}{2\zeta_0} \tag{3.5}$$

where $\xi_4 \neq 0, \zeta_0 \neq 0$. Consecutively, resolving the algebraic equation system (2.9) satisfies

$$\begin{aligned}
\xi_0 = \xi_0, \xi_1 &= \frac{(4\alpha\xi_4 + 3\beta\xi_3\tau_1)\left(8\alpha^2\xi_4^2 - 6\alpha\beta\xi_3\xi_4\tau_1 - 9\beta^2(\xi_3^2 - 4\xi_2\xi_4)\tau_1^2\right)}{216\beta^3\xi_4^2\tau_1^3},\\
\xi_2 = \xi_2, \xi_3 = \xi_3, \xi_4 = \xi_4, \zeta_0 &= \frac{1}{288}\left(-24\xi_2 + \frac{9\xi_3^2}{\xi_4} - \frac{16(\alpha^2 - 3\beta)\xi_4}{\beta^2\tau_1^2}\right),\\
\tau_0 = \frac{\alpha}{3\beta} + \frac{\xi_3\tau_1}{4\xi_4}, \tau_1 = \tau_1, v_0 &= \frac{-4\sqrt{3\beta n\xi_4}}{\sqrt{-16(\alpha^2 - 3\beta)k\xi_4^2 + 3\beta^2 k(3\xi_3^2 - 8\xi_2\xi_4)\tau_1^2}}.
\end{aligned} \tag{3.6}$$

Embedding these corollaries into Eqs. (2.5) and (2.10), we acquire

$$\pm(\eta - \eta_0) = A\int \frac{d\Gamma}{\sqrt{\frac{\xi_0}{\xi_4} + \frac{\xi_1}{\xi_4}\Gamma + \frac{\xi_2}{\xi_4}\Gamma^2 + \frac{\xi_3}{\xi_4}\Gamma^3 + \Gamma^4}}, \tag{3.7}$$

where $A = \sqrt{\frac{\zeta_0}{\xi_4}} = \sqrt{\frac{1}{288}\left(\frac{-24\xi_2}{\xi_4} + \frac{9\xi_3^2}{\xi_4^2} - \frac{16\alpha^2 - 48\beta}{\beta^2\tau_1^2}\right)}$.
Integrating Eq. (3.7), we gain the solutions of Eq. (1.1) as follows:

$$\pm(\eta - \eta_0) = \frac{-A}{\Gamma - \alpha_1}, \tag{3.8}$$

$$\pm(\eta - \eta_0) = \frac{2A}{\alpha_1 - \alpha_2}\sqrt{\frac{\Gamma - \alpha_2}{\Gamma - \alpha_1}}, \alpha_2 > \alpha_1, \tag{3.9}$$

$$\pm(\eta - \eta_0) = \frac{A}{\alpha_1 - \alpha_2} ln\left|\frac{\Gamma - \alpha_1}{\Gamma - \alpha_2}\right|, \alpha_1 > \alpha_2, \tag{3.10}$$

$$\pm(\eta - \eta_0) = \frac{2A}{\sqrt{(\alpha_1 - \alpha_3)(\alpha_1 - \alpha_2)}} ln\left|\frac{\sqrt{(\alpha_1 - \alpha_3)(\Gamma - \alpha_2)} - \sqrt{(\alpha_1 - \alpha_2)(\Gamma - \alpha_3)}}{\sqrt{(\alpha_1 - \alpha_3)(\Gamma - \alpha_2)} + \sqrt{(\alpha_1 - \alpha_2)(\Gamma - \alpha_3)}}\right|, \alpha_1 > \alpha_2 > \alpha_3, \tag{3.11}$$

$$\pm(\eta - \eta_0) = \frac{2A}{\sqrt{(\alpha_1 - \alpha_3)(\alpha_2 - \alpha_4)}} F(\varphi, l), \alpha_4 < \alpha_3 < \alpha_2 < \alpha_1, \tag{3.12}$$

where $F(\varphi, l) = \int_0^{\varphi} \frac{d\psi}{\sqrt{1 - l^2 sin^2\psi}}, \varphi = arcsin\sqrt{\frac{(\Gamma - \alpha_1)(\alpha_2 - \alpha_4)}{(\Gamma - \alpha_2)(\alpha_1 - \alpha_4)}}, l^2 = \frac{(\alpha_2 - \alpha_3)(\alpha_1 - \alpha_4)}{(\alpha_1 - \alpha_3)(\alpha_2 - \alpha_4)}$. Moreover, α_1 and α_2 are the roots of the polynomial equation

$$\Gamma^4 + \frac{\xi_3}{\xi_4}\Gamma^3 + \frac{\xi_2}{\xi_4}\Gamma^2 + \frac{\xi_1}{\xi_4}\Gamma + \frac{\xi_0}{\xi_4} = 0. \tag{3.13}$$

Embedding the solutions (3.8-3.12) into Eq. (2.4), we ascertain the following exact traveling wave solutions of Eq. (1.1), respectively: rational function, hyperbolic function and Jacobi elliptic function.

$$u_1(x, y, t) = \pm\frac{A_1}{\sqrt{k}(x + y - v_0 t)}, \tag{3.14}$$

$$u_2(x, y, t) = \frac{4A^2(\alpha_2 - \alpha_1)\tau_1}{4A^2 - \left[(\alpha_1 - \alpha_2)\left(\sqrt{k}(x + y - v_0 t)\right)\right]^2}, \tag{3.15}$$

$$u_3(x, y, t) = \frac{(\alpha_2 - \alpha_1)\tau_1}{2}\left(1 \pm coth\left[\frac{(\alpha_1 - \alpha_2)}{A}\sqrt{k}(x + y - v_0 t)\right]\right), \tag{3.16}$$

$$u_4(x, y, t) = \frac{A_2}{D + cosh\left[B\left(\sqrt{k}(x + y - v_0 t)\right)\right]}, \tag{3.17}$$

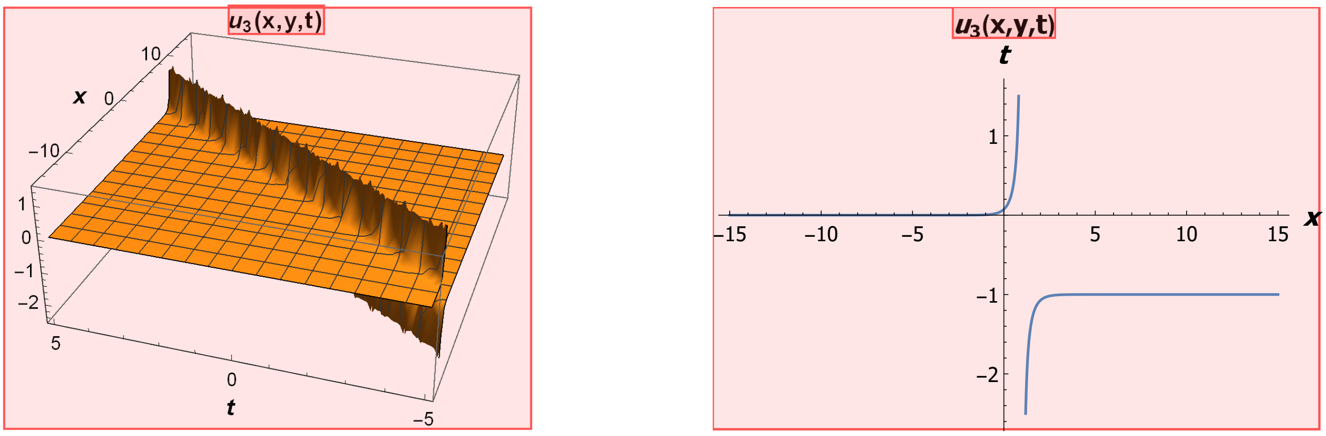

Figure 3.1: Graph of the solution (3.16) is indicated at $k=1, v_0=2, \tau_1=1, \xi_2=3, \xi_3=3, \xi_4=-1, \alpha_1=3, \alpha_2=2, \alpha=2, \beta=4, y=1, -15\leq x\leq 15, -5\leq t\leq 5$ and the second graph denotes the exact solution of Eq. (3.16) for $t=1$ with these values and x range.

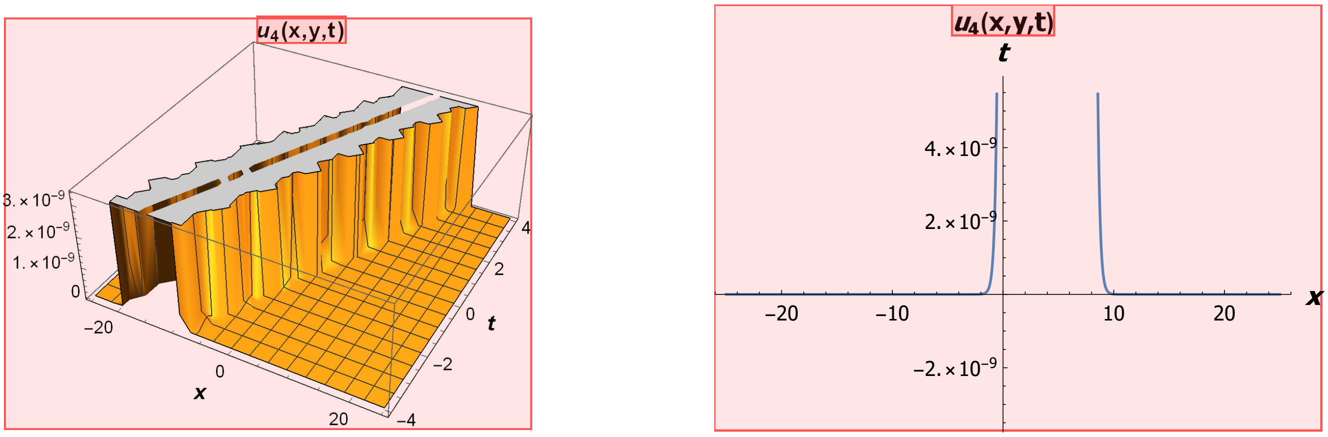

Figure 3.2: Graph of the solution (3.17) is indicated at $k=4, v_0=3, \tau_1=2, \xi_2=5, \xi_3=4, \xi_4=-2, \alpha=1, \alpha_1=1, \beta=2, \alpha_2=2, \alpha_3=3, y=2, -25\leq x\leq 25, -4\leq t\leq 4$ and the second graph denotes the exact solution of Eq. (3.17) for $t=2$ with these values and x range.

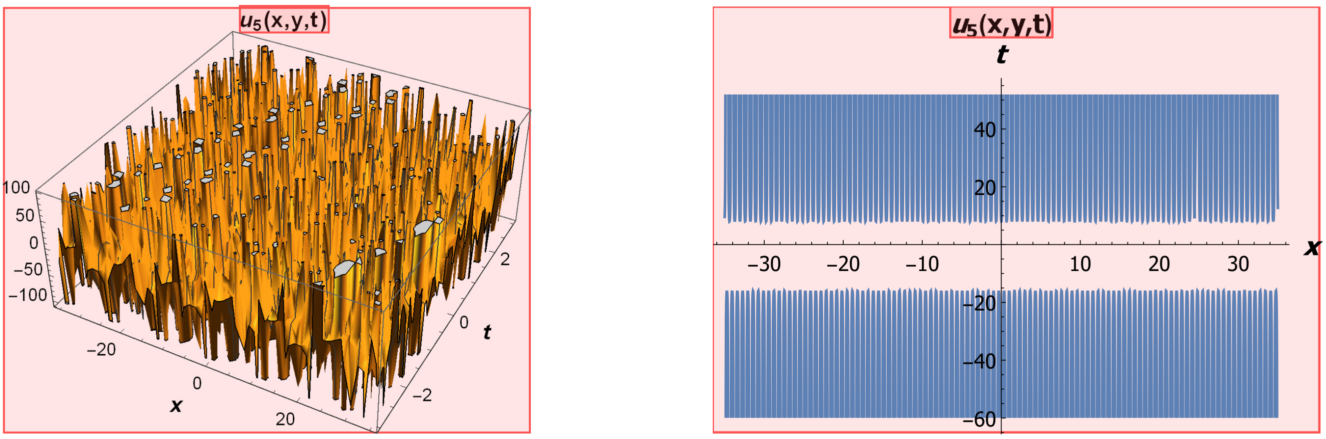

Figure 3.3: Graph of the solution (3.18) is indicated at $k=9, v_0=5, \tau_1=-2, \xi_2=1, \xi_3=2, \xi_4=-1, y=3, \alpha=3, \alpha_1=1, \beta=6, \alpha_2=2, \alpha_3=3, \alpha_4=4, -35\leq x\leq 35, -3\leq t\leq 3$ and the second graph denotes the exact solution of Eq. (3.18) for $t=1$ with these values and x range.

$$u_5(x,y,t)=\frac{A_3}{\left(M+Nsn^2(\varphi,l)\right)}, \tag{3.18}$$

where $A_1=\tau_1 A, A_2=\frac{2\tau_1(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}{\alpha_3-\alpha_2}, A_3=\left(2\tau_1(\alpha_4-\alpha_2)(\alpha_1-\alpha_3)\right),$
$B=\frac{\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}}{A}, D=\frac{2\alpha_1-\alpha_2-\alpha_3}{\alpha_3-\alpha_2}, l^2=\frac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}, M=(\alpha_4-\alpha_2),$
$N=(\alpha_1-\alpha_4), \varphi=\pm\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2A}\left(\sqrt{k}(x+y-v_0t)\right).$

Remark 1. If the notation $l \to 1$ then the solution (3.18) can be turned into the hyperbolic function solution of the (2+1)-dimensional NETLE

$$u_6(x,y,t) = \frac{A_3}{M + Ntanh^2\left[\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2A}\left(\sqrt{k}(x+y-v_0t)\right)\right]}, \tag{3.19}$$

where $\alpha_3 = \alpha_4$.

Remark 2. If the modulus $l \to 0$ then the solution (3.18) can be abbreviated to periodic wave solution

$$u_7(x,y,t) = \frac{A_3}{M + Nsin^2\left[\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2A}\left(\sqrt{k}(x+y-v_0t)\right)\right]}, \tag{3.20}$$

where $\alpha_2 = \alpha_3$.

Case 2: If we choose $\varepsilon = 1, \delta = 1$ and $\vartheta = 5$ we have,

$$(u')^2 = \frac{\tau_1^2\left(\xi_5\Gamma^5 + \xi_4\Gamma^4 + \xi_3\Gamma^3 + \xi_2\Gamma^2 + \xi_1\Gamma + \xi_0\right)}{\zeta_0 + \zeta_1\Gamma} \tag{3.21}$$

$$u'' = \frac{\tau_1\left(\zeta_0 + \zeta_1\Gamma\right)\left(5\xi_5\Gamma^4 + 4\xi_4\Gamma^3 + 3\xi_3\Gamma^2 + 2\xi_2\Gamma + \xi_1\right) - \zeta_1\left(\xi_5\Gamma^5 + \xi_4\Gamma^4 + \xi_3\Gamma^3 + \xi_2\Gamma^2 + \xi_1\Gamma + \xi_0\right)}{2\left(\zeta_0 + \zeta_1\Gamma\right)^2} \tag{3.22}$$

where $\xi_5 \neq 0, \zeta_1 \neq 0$. Consequtively, resolving the algebraic equation system (2.9) yields

$$\begin{aligned}
\xi_0 = \xi_0, \xi_1 &= \frac{-72\beta\zeta_0^3\tau_0\tau_1^2 - 4\zeta_1^2\xi_0 p\tau_1^2 + 3\beta\zeta_0\zeta_1\xi_0\tau_1^3 + 12\zeta_0^2\tau_0(\xi_4 r + 8\zeta_1 p\tau_1)}{\zeta_0\tau_1^2\left(-4\zeta_1 p + 3\beta\zeta_0\tau_1\right)},\\
\xi_2 &= \frac{6\left(16\zeta_1^2\tau_0 p\tau_1 + \zeta_0\tau_1\left(\xi_4 s - 6\beta\zeta_0\tau_1^2\right) + 2\zeta_1\left(\xi_4\tau_0 r + 2\zeta_0(2\alpha - 9\beta\tau_0)\tau_1^2\right)\right)}{\tau_1^2\left(-4\zeta_1 p + 3\beta\zeta_0\tau_1\right)},\\
\xi_3 &= \frac{6\zeta_1\xi_4 s + 4\left(12\zeta_1^2 - \zeta_0\xi_4\right)p\tau_1 - 36\beta\zeta_0\zeta_1\tau_1^2}{\tau_1\left(-4\zeta_1 p + 3\beta\zeta_0\tau_1\right)}, \xi_4 = \xi_4,\\
\xi_5 &= \frac{3\beta\zeta_1\xi_4\tau_1}{-4\zeta_1 p + 3\beta\zeta_0\tau_1}, v_0 = -\frac{\sqrt{\left(u_0^2K_1 + \omega_0^2K_2\right)\xi_4}}{\sqrt{2\left(k\tau_1\left(-4\zeta_1 p + 3\beta\zeta_0\tau_1\right)\right)}},
\end{aligned} \tag{3.23}$$

where $p = \alpha - 3\beta\tau_0, r = 1 - \alpha\tau_0 + \beta\tau_0^2, s = 1 - 2\alpha\tau_0 + 3\beta\tau_0^2$. Embedding these corollaries into Eqs. (2.5) and (2.10), we acquire

$$\pm(\eta - \eta_0) = \sqrt{\frac{\zeta_1(4\tau_0 + 2) + \zeta_0\tau_1}{\xi_4\tau_1}}\sqrt{\int\frac{\Gamma + \frac{\zeta_0}{\zeta_1}}{\Gamma^5\frac{\xi_4}{\xi_5}\Gamma^4 + \frac{\xi_3}{\xi_5}\Gamma^3 + \frac{\xi_2}{\xi_5}\Gamma^2 + \frac{\xi_1}{\xi_5}\Gamma + \frac{\xi_0}{\xi_5}}d\Gamma}, \tag{3.24}$$

Integrating Eq. (3.24), we get the following exact approximate solutions to the Eq. (3.2). When $\phi(\Gamma) = (\Gamma - \alpha_1)^5$, we have

$$\pm(\eta - \eta_0) = -\frac{2A_4}{3\sqrt{\zeta_1}(\zeta_0 + \zeta_1\alpha_1)}\left(\frac{\zeta_0 + \zeta_1\Gamma}{\Gamma - \alpha_1}\right)^{\frac{3}{2}}, \tag{3.25}$$

If we take $\phi(\Gamma) = (\Gamma - \alpha_1)^4(\Gamma - \alpha_2)$ and $\alpha_1 > \alpha_2$, then we get,

$$\pm(\eta - \eta_0) = -\frac{A_4}{\alpha_1 - \alpha_2}\left[\frac{(\zeta_0 + \zeta_1\alpha_2)}{2\sqrt{\zeta_1(\alpha_1 - \alpha_2)(\zeta_0 + \zeta_1\alpha_1)}}ln|K(\Gamma)| + \frac{1}{\Gamma - \alpha_1}\sqrt{\frac{(\zeta_0 + \zeta_1\Gamma)(\Gamma - \alpha_2)}{\zeta_1}}\right], \tag{3.26}$$

where

$$K(\Gamma) = \frac{\Gamma - \alpha_1}{(\zeta_0 + 2\zeta_1\alpha_1 - \zeta_1\alpha_2)\Gamma + \zeta_0(\alpha_1 - 2\alpha_2) - \zeta_1\alpha_2\alpha_1 + 2\sqrt{(\zeta_0 + \zeta_1\Gamma)}(\zeta_0 + \zeta_1\alpha_1)(\Gamma - \alpha_2)(\alpha_1 - \alpha_2)}, \tag{3.27}$$

when $\phi(\Gamma) = (\Gamma - \alpha_1)^3(\Gamma - \alpha_2)^2$ and $\alpha_1 > \alpha_2$, we obtain

$$\pm(\eta - \eta_0) = -\frac{2A_4}{\alpha_1 - \alpha_2}\left|\sqrt{\frac{(\zeta_0 + \zeta_1\Gamma)}{(\zeta_1(\Gamma - \alpha_1))}} + \sqrt{\frac{(\zeta_0 + \zeta_1\alpha_2)}{\zeta_1(\alpha_1 - \alpha_2)}arctan\left(\sqrt{\frac{(\Gamma - \alpha_1)(\zeta_0 + \zeta_1\alpha_2)}{(\alpha_1 - \alpha_2)((\zeta_0 + \zeta_1\Gamma))}}\right)}\right|, \tag{3.28}$$

If we take $\phi(\Gamma) = (\Gamma - \alpha_1)^2(\Gamma - \alpha_2)^2(\Gamma - \alpha_3)$ and $\alpha_1 > \alpha_2 > \alpha_3$, then we get,

$$\pm(\eta - \eta_0) = -\frac{A_4}{(\alpha_1 - \alpha_3)\sqrt{\zeta_1}}\left[Yln|P(\Gamma)| + Zln|R(\Gamma)|\right], \tag{3.29}$$

where

$$Y=\sqrt{\frac{\zeta_0+\zeta_1\alpha_2}{\alpha_2-\alpha_3}},P(\Gamma)=\frac{\alpha_2-\Gamma}{2\sqrt{(\zeta_0+\zeta_1\Gamma)(\zeta_0+\zeta_1\alpha_2)(\Gamma-\alpha_3)(\alpha_2-\alpha_3)}+\zeta_0(\alpha_2-2\alpha_3)-\zeta_1\alpha_2\alpha_3+(\zeta_0+2\zeta_1\alpha_2-\zeta_1\alpha_3)\Gamma},$$
$$Z=\sqrt{\frac{\zeta_0+\zeta_1\alpha_1}{\alpha_1-\alpha_3}},R(\Gamma)=\frac{2\sqrt{(\zeta_0+\zeta_1\Gamma)(\zeta_0+\zeta_1\alpha_1)(\Gamma-\alpha_3)(\alpha_1-\alpha_3)}+\zeta_0(\alpha_1-2\alpha_3)-\zeta_1\alpha_1\alpha_3+(\zeta_0+2\zeta_1\alpha_1-\zeta_1\alpha_3)\Gamma}{\Gamma-\alpha_2} \tag{3.30}$$

when $\phi(\Gamma)=(\Gamma-\alpha_1)^3(\Gamma-\alpha_2)(\Gamma-\alpha_3)$ and $\alpha_1>\alpha_2>\alpha_3$, then we obtain

$$\pm(\eta-\eta_0)=-\frac{2A_4}{\alpha_1-\alpha_3}\sqrt{\frac{(\zeta_0+\zeta_1\alpha_3)}{\zeta_1(\alpha_1-\alpha_2)}}E(\varphi,l), \tag{3.31}$$

where

$$\varphi=arcsin\sqrt{\frac{(\Gamma-\alpha_3)(\alpha_2-\alpha_1)}{(\Gamma-\alpha_1)(\alpha_2-\alpha_3)}},l^2=\frac{(\zeta_0+\zeta_1\alpha_1)(\alpha_3-\alpha_2)}{(\zeta_0+\zeta_1\alpha_3)(\alpha_1-\alpha_2)}, \tag{3.32}$$

If we take $\phi(\Gamma)=(\Gamma-\alpha_1)^2(\Gamma-\alpha_2)(\Gamma-\alpha_3)(\Gamma-\alpha_4)$ and $\alpha_1>\alpha_2>\alpha_3$, then we get,

$$\pm(\eta-\eta_0)=-\frac{2A_4(\alpha_2-\alpha_4)}{(\alpha_1-\alpha_2)\sqrt{\zeta_1(\alpha_2-\alpha_3)(\zeta_0+\zeta_1\alpha_4)}}\left(\frac{\zeta_0+\zeta_1\Gamma}{\alpha_1-\alpha_4}\pi(\varphi,n,l)-\frac{\zeta_0+\zeta_1\alpha_2}{\alpha_2-\alpha_4}F(\varphi,l)\right), \tag{3.33}$$

where

$$A_4=\sqrt{\frac{\zeta_1(4\tau_0+2)+\zeta_0\tau_1}{\xi_4\tau_1}},\varphi=arcsin\sqrt{\frac{(\Gamma-\alpha_4)(\alpha_3-\alpha_2)}{(\Gamma-\alpha_2)(\alpha_3-\alpha_4)}},l^2=\frac{(\zeta_0+\zeta_1\alpha_2)(\alpha_4-\alpha_3)}{(\zeta_0+\zeta_1\alpha_4)(\alpha_2-\alpha_3)},n=-\frac{(\alpha_1-\alpha_2)(\alpha_3-\alpha_4)}{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}. \tag{3.34}$$

Remark 3. A number of solutions of Eq. (1.1) were obtained by applying ETEM and these found solutions were controlled in Mathematica 12. Obtained Equation (3.16) in this work is similar to the solution (25) offered by Kayum et al. (2020). Besides, other solutions of Eq. (1.1) are new.

3.2. Implementation of the BBMP equation

In order to find travelling wave solutions of Eq. (1.2), we take the transformation $u(x,t)=U(\eta),\eta=x-\lambda t$, we acquire

$$-\lambda U_\eta+aU_\eta+bU^qU_\eta-\lambda cU_{\eta\eta\eta}=0, \tag{3.35}$$

Taking into consideration the conversion

$$U=V^{\frac{1}{q}}, \tag{3.36}$$

Eq. (48) turns into the formula

$$-cq(q+1)\lambda VV_{\eta\eta}+c(q^2-1)\lambda V_\eta^2+q^2(1+q)(a-\lambda)V^2+bq^2V^3=0,q\neq0,q\neq\pm1, \tag{3.37}$$

Embedding Eqs. (2.6) and (2.7) into Eq. (3.3), and utilizing the balance principle, we gain

$$\vartheta=\varepsilon+\delta+2, \tag{3.38}$$

After the above solution steps, the following situations are obtained:

Case 1: If we choose $\varepsilon=0,\delta=1$ and $\vartheta=3$ we have,

$$(v')^2=\frac{\tau_1^2\left(\xi_3\Gamma^3+\xi_2\Gamma^2+\xi_1\Gamma+\xi_0\right)}{\zeta_0},v''=\frac{\tau_1\left(3\xi_3\Gamma^2+2\xi_2\Gamma+\xi_1\right)}{2\zeta_0} \tag{3.39}$$

where $\xi_3\neq0,\zeta_0\neq0$. Respectively, solving the algebraic equation system (2.9) yields

$$\xi_0=\xi_0,\xi_1=\xi_1,\xi_2=\frac{(-4+q)q\xi_1^2}{4(-1+q)^2\xi_0},\xi_3=-\frac{q^2\xi_1^3}{4(-1+q)^3\xi_0^2},$$
$$\zeta_0=-\frac{c(q+2)\xi_1^2(a+aq+b\tau_0)}{4b(q-1)^2q\xi_0\tau_0},\tau_0=\tau_0,\tau_1=\frac{q\xi_1\tau_0}{2(q-1)\xi_0},\lambda=a+\frac{b\tau_0}{q+1}. \tag{3.40}$$

Embedding these corollaries into Eqs. (2.5) and (2.10), we gain

$$\pm(\eta-\eta_0)=\sqrt{A_8}\int\frac{d\Gamma}{\sqrt{\Gamma^3+\frac{\xi_2}{\xi_3}\Gamma^2+\frac{\xi_1}{\xi_3}\Gamma+\frac{\xi_0}{\xi_3}}}, \tag{3.41}$$

where $A_8=\frac{\xi_0}{\xi_3}=\frac{\xi_0 c(q-1)(q+2)(a+aq+b\tau_0)}{bq^3\tau_0\xi_1}$.
Integrating Eq. (3.41), we achieve the solutions of Eq. (1.2) as follows:

$$\pm(\eta-\eta_0)=-2\sqrt{A_8}\frac{1}{\sqrt{\Gamma-\alpha_1}}, \tag{3.42}$$

$$\pm(\eta-\eta_0)=2\sqrt{\frac{A_8}{\alpha_2-\alpha_1}}arctan\sqrt{\frac{\Gamma-\alpha_2}{\alpha_2-\alpha_1}},\alpha_2>\alpha_1, \tag{3.43}$$

$$\pm(\eta-\eta_0)=\sqrt{\frac{A_8}{\alpha_1-\alpha_2}}ln\left|\frac{\sqrt{\Gamma-\alpha_2}-\sqrt{\alpha_1-\alpha_2}}{\sqrt{\Gamma-\alpha_2}+\sqrt{\alpha_1-\alpha_2}}\right|,\alpha_1>\alpha_2, \tag{3.44}$$

$$\pm(\eta-\eta_0)=2\sqrt{\frac{A_8}{\alpha_1-\alpha_3}}F(\varphi,l),\alpha_1>\alpha_2>\alpha_3, \tag{3.45}$$

where

$$F(\varphi,l)=\int_0^{\varphi}\frac{d\psi}{1-l^2sin^2\psi},\varphi=arcsin\sqrt{\frac{\Gamma-\alpha_3}{\alpha_2-\alpha_3}},l^2=\frac{\alpha_2-\alpha_3}{\alpha_1-\alpha_3}. \tag{3.46}$$

Also, α_1,α_2 and α_3 are the roots of the polynomial equation

$$\Gamma^3+\frac{\xi_2}{\xi_3}\Gamma^2+\frac{\xi_1}{\xi_3}\Gamma+\frac{\xi_0}{\xi_3}=0, \tag{3.47}$$

Embedding the solutions (3.42-3.45) into Eq. (2.4) and Eq. (3.36), we ascertain the following exact traveling wave solutions of Eq. (1.2), respectively: rational function, hyperbolic function and Jacobi elliptic function.

$$u(x,t)=\left[\tau_0+\tau_1\alpha_1+\frac{4\tau_1A_8}{\left(x-\left(a+\frac{b\tau_0}{q+1}\right)t-\eta_0\right)^2}\right]^{\frac{1}{q}}, \tag{3.48}$$

$$u(x,t)=\left[\tau_0+\tau_1\alpha_1+\tau_1(\alpha_2-\alpha_1)tanh^2\left(\frac{1}{2}\sqrt{\frac{\alpha_1-\alpha_2}{A_8}}\left(x-\left(a+\frac{b\tau_0}{q+1}\right)t-\eta_0\right)\right)\right]^{\frac{1}{q}}, \tag{3.49}$$

$$u(x,t)=\left[\tau_0+\tau_1\alpha_1+\tau_1(\alpha_1-\alpha_2)cosech^2\left(\frac{1}{2}\sqrt{\frac{\alpha_1-\alpha_2}{A_8}}\left(x-\left(a+\frac{b\tau_0}{q+1}\right)t\right)\right)\right]^{\frac{1}{q}}, \tag{3.50}$$

and

$$u(x,t)=\left[\tau_0+\tau_1\alpha_3+\tau_1(\alpha_2-\alpha_3)sn^2\left(\pm\frac{1}{2}\sqrt{\frac{\alpha_1-\alpha_3}{A_8}}\left(x-\left(a+\frac{b\tau_0}{q+1}\right)t-\eta_0\right),\frac{\alpha_2-\alpha_3}{\alpha_1-\alpha_3}\right)\right]^{\frac{1}{q}}, \tag{3.51}$$

If we take $\tau_0=-\tau_1\alpha_1$ and $\eta_0=0$ for simplicity, then the solutions (3.48)-(3.50) can reduce to rational function solution

$$u(x,t)=\left(\frac{2\sqrt{\tilde{A}}}{x-\left(a+\frac{b\tau_0}{q+1}\right)t}\right)^{\frac{2}{q}}, \tag{3.52}$$

1-soliton solution

$$u(x,t)=\frac{A_9}{cosh^{\frac{2}{q}}\left[B_2\left(x-\left(a+\frac{b\tau_0}{q+1}\right)t\right)\right]}, \tag{3.53}$$

singular soliton solution

$$u(x,t)=\frac{A_{10}}{sinh^{\frac{2}{q}}\left[B_2\left(x-\left(a+\frac{b\tau_0}{q+1}\right)t\right)\right]}, \tag{3.54}$$

where
$\tilde{A}=\tau_1A_8,A_9=(\tau_1(\alpha_2-\alpha_1))^{\frac{1}{q}},A_{10}=(\tau_1(\alpha_1-\alpha_2))^{\frac{1}{q}}\,B_2=\pm\frac{1}{2}\sqrt{\frac{\alpha_1-\alpha_2}{A_8}},\lambda=a+\frac{b\tau_0}{q+1}$, Here, A_9 and A_{10} are the amplitudes of the solitons, while λ is the velocity and B_2 is the inverse width of the solitons. So, we can remark the solitons exist for $\tau_1>0$. Also, if we take $\tau_0=-\tau_1\alpha_3$ and $\eta_0=0$ the Jacobi elliptic function solution (3.51) can be written as

$$u_i(x,t)=A_{11}sn^{\frac{2}{q}}\left[B_i\left(x-\left(a+\frac{b\tau_0}{q+1}\right)t\right),\frac{\alpha_2-\alpha_3}{\alpha_1-\alpha_3}\right], \tag{3.55}$$

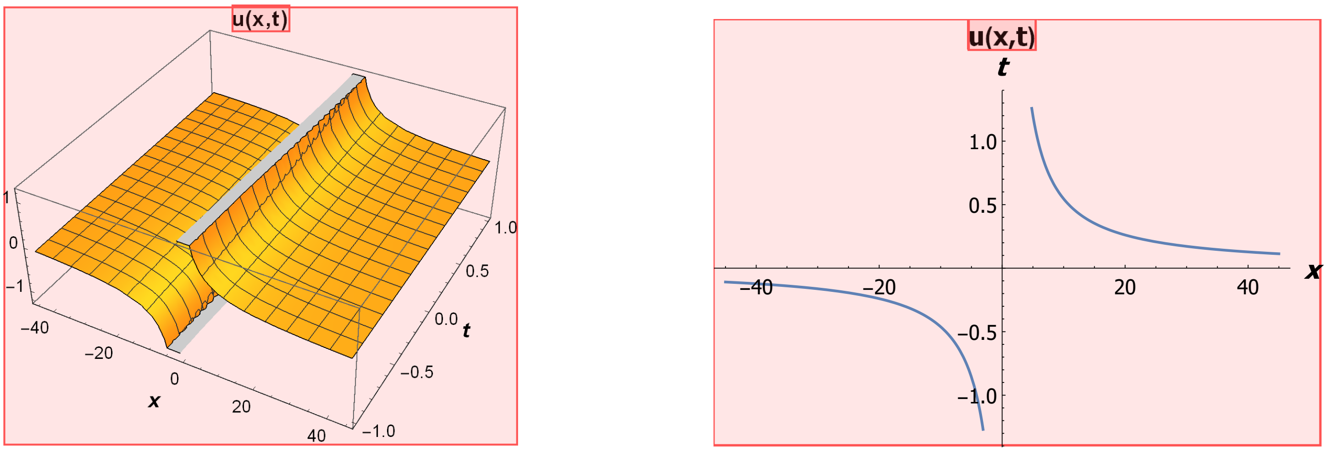

Figure 3.4: Graph of the solution (3.52) is indicated at $\tau_0 = 1, \tau_1 = 3, a = 1, b = 2, c = -1, \xi_0 = 5, \xi_1 = -3, q = 2, -45 \leq x \leq 45$, $-1 \leq t \leq 1$ and the second graph denotes the exact solution of Eq. (3.52) for $t = 0.5$ with these values and x range.

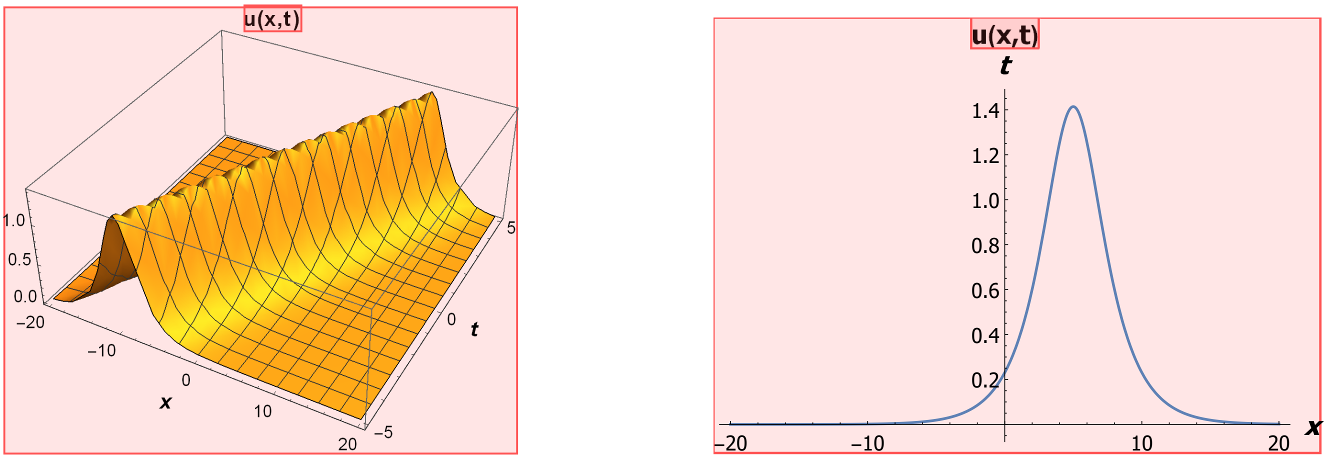

Figure 3.5: Graph of the solution (3.53) is indicated at $\tau_0 = -3, \tau_1 = 2, a = 5, b = 3, c = 1, \xi_0 = 3, \xi_1 = 1, \alpha_1 = 1, \alpha_2 = 2, q = 2, -20 \leq x \leq 20$, $-5 \leq t \leq 5$ and the second graph denotes the exact solution of Eq. (3.53) for $t = 2.5$ with these values and x range.

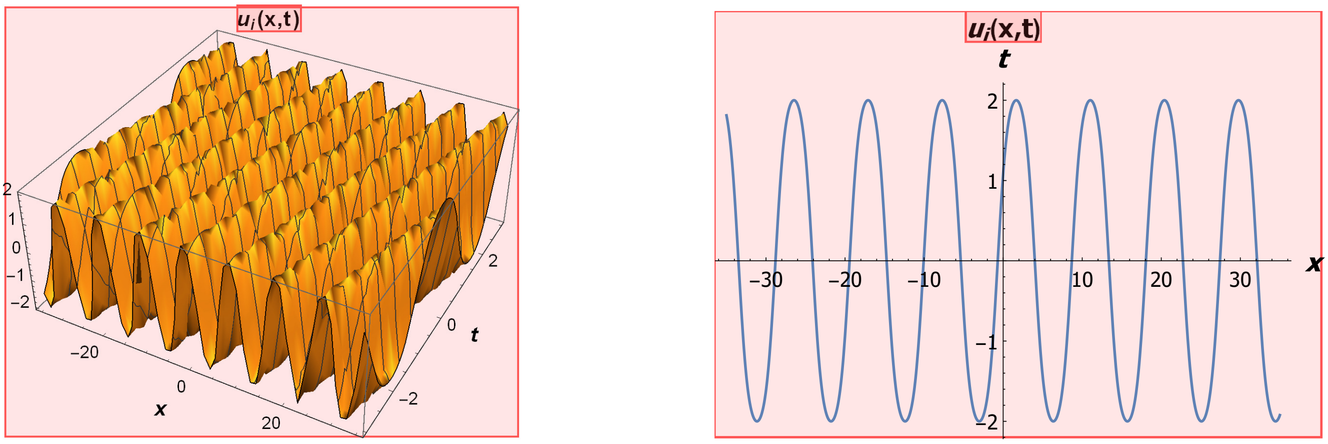

Figure 3.6: Graph of the solution (3.54) is indicated at $\tau_0 = -2, \tau_1 = 2, a = 2, b = -1, c = 3, \xi_0 = 1, \xi_1 = 6, \alpha_1 = 4, \alpha_2 = 3, q = 2, -35 \leq x \leq 35$, $-3 \leq t \leq 3$ and the second graph denotes the exact solution of Eq. (3.54) for $t = 1.5$ with these values and x range.

where $A_{11} = (\tau_1(\alpha_2 - \alpha_3))^{\frac{1}{q}}$ and $B_i = \frac{(-1)^i}{2}\sqrt{\frac{\alpha_1 - \alpha_3}{A_8}}, (i = 1, 2)$.

Remark 1: When the modulus $l \to 1$, the solution (3.55) can be transformed dark soliton solutions of BBMP equation

$$u_i(x,t) = A_{11} tanh^{\frac{2}{q}} \left[B_i \left(x - \left(a + \frac{b\tau_0}{q+1} \right) t \right) \right], \tag{3.56}$$

where $\alpha_1 = \alpha_2$ and $\lambda = \left(a + \frac{b\tau_0}{q+1} \right)$ symbolizes the speed of the dark soliton.

Case 2: If we choose $\varepsilon=0,\delta=2$ and $\vartheta=4$ we have,

$$(v')^2=\frac{(\tau_1+2\tau_2\Gamma)^2\left(\xi_4\Gamma^4+\xi_3\Gamma^3+\xi_2\Gamma^2+\xi_1\Gamma+\xi_0\right)}{\zeta_0},$$
$$v''=\frac{(\tau_1+2\tau_1\Gamma)\left(4\xi_4\Gamma^3+2\xi_3\Gamma^2+2\xi_2\Gamma+\xi_1\right)}{2\zeta_0}+\frac{2\tau_2\left(\xi_4\Gamma^4+\xi_3\Gamma^3+\xi_2\Gamma^2+\xi_1\Gamma+\xi_0\right)}{\zeta_0}, \tag{3.57}$$

where $\xi_4\neq 0,\zeta_0\neq 0$. Consecutively, resolving the algebraic equation system (2.9) provides

$$\xi_0=\frac{-bq^2\zeta_0\tau_1^4+4c\xi_1\tau_1\tau_2\left(b\tau_1^2+2a(q+1)(q+2)\tau_2\right)}{32c\tau_2^2\left(b\tau_1^2+a(q+1)(q+2)\tau_2\right)},\xi_1=\xi_1,$$
$$\xi_2=\frac{bq^2\zeta_0\tau_1^3+2c\xi_1\tau_2\left(3b\tau_1^2+a(q+1)(q+2)\tau_2\right)}{2c\tau_1\left(b\tau_1^2+a(q+1)(q+2)\tau_2\right)},$$
$$\xi_3=\frac{b\tau_2\left(q^2\zeta_0\tau_1+4c\xi_1\tau_2\right)}{\left(bc\tau_1^2+ac(q+1)(q+2)\tau_2\right)},\xi_4=\frac{b\tau_2^2\left(q^2\zeta_0\tau_1+4c\xi_1\tau_2\right)}{2c\tau_1\left(b\tau_1^2+a(q+1)(q+2)\tau_2\right)},$$
$$\tau_0=\frac{\tau_1^2}{4\tau_2},\tau_1=\tau_1,\lambda=\frac{q^2\zeta_0\tau_1\left(b\tau_1^2+a(q+1)(q+2)\tau_2\right)}{(q+1)(q+2)\tau_2\left(q^2\zeta_0\tau_1+4c\xi_1\tau_2\right)}. \tag{3.58}$$

Embedding these corollaries into Eqs. (2.5) and (2.10), we get

$$\pm(\eta-\eta_0)=\sqrt{\frac{\zeta_0 2c\tau_1\left(b\tau_1^2+a(q+1)(q+2)\tau_2\right)}{b\tau_2^2\left(q^2\zeta_0\tau_1+4c\xi_1\tau_2\right)}}\int\frac{d\Gamma}{\sqrt{\Gamma^4+\frac{\xi_3}{\xi_4}\Gamma^3+\frac{\xi_2}{\xi_4}\Gamma^2+\frac{\xi_1}{\xi_4}\Gamma+\frac{\xi_0}{\xi_4}}}, \tag{3.59}$$

Integrating Eq. (3.59), we obtain the following exact solutions of Eq. (1.2).

$$\pm(\eta-\eta_0)=-\frac{A_{12}}{\Gamma-\alpha_1}, \tag{3.60}$$

$$\pm(\eta-\eta_0)=2\frac{A_{12}}{\alpha_1-\alpha_2}\sqrt{\frac{\Gamma-\alpha_2}{\Gamma-\alpha_1}},\alpha_1>\alpha_2, \tag{3.61}$$

$$\pm(\eta-\eta_0)=\frac{A_{12}}{\alpha_1-\alpha_2}ln\left|\frac{\Gamma-\alpha_1}{\Gamma-\alpha_2}\right|, \tag{3.62}$$

$$\pm(\eta-\eta_0)=\frac{2A_{12}}{\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}}ln\left|\frac{\sqrt{(\Gamma-\alpha_2)(\alpha_1-\alpha_3)}-\sqrt{(\Gamma-\alpha_3)(\alpha_1-\alpha_2)}}{\sqrt{(\Gamma-\alpha_2)(\alpha_1-\alpha_3)}+\sqrt{(\Gamma-\alpha_3)(\alpha_1-\alpha_2)}}\right|,\alpha_1>\alpha_2>\alpha_3, \tag{3.63}$$

$$\pm(\eta-\eta_0)=\frac{2A_{12}}{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}F(\varphi,l),\alpha_1>\alpha_2>\alpha_3>\alpha_4, \tag{3.64}$$

where

$$A_{12}=\sqrt{\frac{\zeta_0 2c\tau_1\left(b\tau_1^2+a(q+1)(q+2)\tau_2\right)}{b\tau_2^2\left(q^2\zeta_0\tau_1+4c\xi_1\tau_2\right)}},\varphi=arcsin\sqrt{\frac{(\Gamma-\alpha_1)(\alpha_2-\alpha_4)}{(\Gamma-\alpha_2)(\alpha_1-\alpha_4)}},l^2=\frac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}. \tag{3.65}$$

Also, $\alpha_1,\alpha_2,\alpha_3$ and α_4 are the roots of the polynomial equation

$$\Gamma^4+\frac{\xi_3}{\xi_4}\Gamma^3+\frac{\xi_2}{\xi_4}\Gamma^2+\frac{\xi_1}{\xi_4}\Gamma+\frac{\xi_0}{\xi_4}=0, \tag{3.66}$$

Embedding the solutions (3.60)-(3.64) into (2.4) and (3.36), we get

$$u(x,t)=\left[\tau_0+\tau_1\alpha_1\pm\frac{\tau_1A_{12}}{x-\lambda t-\eta_0}+\tau_2\left(\alpha_1\pm\frac{A_{12}}{x-\lambda t-\eta_0}\right)^2\right]^{\frac{1}{q}}, \tag{3.67}$$

$$u(x,t)=\left[\tau_0+\tau_1\alpha_1+\frac{4A_{12}^2(\alpha_2-\alpha_1)\tau_1}{4A_{12}^2-\left[(\alpha_1-\alpha_2)\left(x-\frac{q^2\zeta_0\tau_1\left(b\tau_1^2+a(q+1)(q+2)\tau_2\right)}{(q+1)(q+2)\tau_2(q^2\zeta_0\tau_1+4c\xi_1\tau_2)}t-\eta_0\right)\right]^2}+\right.$$
$$\left.\tau_2\left(\alpha_1+\frac{4A_{12}^2(\alpha_2-\alpha_1)}{4A_{12}^2-\left[(\alpha_1-\alpha_2)\left(x-\frac{q^2\zeta_0\tau_1\left(b\tau_1^2+a(q+1)(q+2)\tau_2\right)}{(q+1)(q+2)\tau_2(q^2\zeta_0\tau_1+4c\xi_1\tau_2)}t-\eta_0\right)\right]^2}\right)^2\right]^{\frac{1}{q}}, \tag{3.68}$$

$$u(x,t)=\left[\tau_0+\tau_1\alpha_2+\frac{(\alpha_2-\alpha_1)\tau_1}{exp\left[\frac{(\alpha_1-\alpha_2)}{A_{12}}\left(x-\frac{q^2\zeta_0\tau_1\left(b\tau_1^2+a(q+1)(q+2)\tau_2\right)}{(q+1)(q+2)\tau_2(q^2\zeta_0\tau_1+4c\xi_1\tau_2)}t-\eta_0\right)\right]-1}+\right.$$
$$\left.\tau_2\left(\alpha_2+\frac{(\alpha_2-\alpha_1)}{exp\left[\frac{(\alpha_1-\alpha_2)}{A_{12}}\left(x-\frac{q^2\zeta_0\tau_1\left(b\tau_1^2+a(q+1)(q+2)\tau_2\right)}{(q+1)(q+2)\tau_2(q^2\zeta_0\tau_1+4c\xi_1\tau_2)}t-\eta_0\right)\right]-1}\right)^2\right]^{\frac{1}{q}}, \tag{3.69}$$

$$u(x,t)=\left[\tau_0+\tau_1\alpha_1+\frac{(\alpha_1-\alpha_2)\tau_1}{exp\left[\frac{(\alpha_1-\alpha_2)}{A_{12}}\left(x-\frac{q^2\zeta_0\tau_1\left(b\tau_1^2+a(q+1)(q+2)\tau_2\right)}{(q+1)(q+2)\tau_2(q^2\zeta_0\tau_1+4c\xi_1\tau_2)}t-\eta_0\right)\right]-1}+\right.$$
$$\left.\tau_2\left(\alpha_1+\frac{(\alpha_1-\alpha_2)}{exp\left[\frac{(\alpha_1-\alpha_2)}{A_{12}}\left(x-\frac{q^2\zeta_0\tau_1\left(b\tau_1^2+a(q+1)(q+2)\tau_2\right)}{(q+1)(q+2)\tau_2(q^2\zeta_0\tau_1+4c\xi_1\tau_2)}t-\eta_0\right)\right]-1}\right)^2\right]^{\frac{1}{q}}, \tag{3.70}$$

$$u(x,t)=\left[\tau_0+\tau_1\alpha_1-\frac{2(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)\tau_1}{2\alpha_1-\alpha_2-\alpha_3+(\alpha_3-\alpha_2)cosh\left[\frac{\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}}{A_{12}}(x-\lambda t-\eta_0)\right]}+\right.$$
$$\left.\tau_2\left(\alpha_1-\frac{2(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}{2\alpha_1-\alpha_2-\alpha_3+(\alpha_3-\alpha_2)cosh\left[\frac{\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}}{A_{12}}(x-\lambda t-\eta_0)\right]}\right)^2\right]^{\frac{1}{q}}, \tag{3.71}$$

$$u(x,t)=\left[\tau_0+\tau_1\alpha_2+\frac{(\alpha_1-\alpha_2)(\alpha_4-\alpha_2)\tau_1}{\alpha_4-\alpha_2+(\alpha_1-\alpha_4)sn^2\left[\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2A_{12}}(x-\lambda t-\eta_0),\frac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}\right]}+\right.$$
$$\left.\tau_2\left(\alpha_2+\frac{(\alpha_1-\alpha_2)(\alpha_4-\alpha_2)\tau_1}{\alpha_4-\alpha_2+(\alpha_1-\alpha_4)sn^2\left[\frac{\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2A_{12}}(x-\lambda t-\eta_0),\frac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}\right]}\right)^2\right]^{\frac{1}{q}}, \tag{3.72}$$

where $\lambda=\frac{q^2\zeta_0\tau_1\left(b\tau_1^2+a(q+1)(q+2)\tau_2\right)}{(q+1)(q+2)\tau_2(q^2\zeta_0\tau_1+4c\xi_1\tau_2)}$.
For plainness, if we get $\eta_0=0$, then we can specify the solutions (3.67)-(3.72) as follows:

$$u(x,t)=\left[\sum_{i=0}^{2}\tau_i\left(\alpha_1\pm\frac{A_{12}}{\left(x-\frac{q^2\zeta_0\tau_1\left(b\tau_1^2+a(q+1)(q+2)\tau_2\right)}{(q+1)(q+2)\tau_2(q^2\zeta_0\tau_1+4c\xi_1\tau_2)}t\right)}\right)^i\right]^{\frac{1}{q}}, \tag{3.73}$$

$$u(x,t)=\left[\sum_{i=0}^{2}\tau_i\left(\alpha_1+\frac{4A_{12}^2(\alpha_1-\alpha_2)}{4A_{12}^2-\left[(\alpha_1-\alpha_2)\left(x-\frac{q^2\zeta_0\tau_1\left(b\tau_1^2+a(q+1)(q+2)\tau_2\right)}{(q+1)(q+2)\tau_2(q^2\zeta_0\tau_1+4c\xi_1\tau_2)}t\right)\right]^2}\right)^i\right]^{\frac{1}{q}}, \tag{3.74}$$

$$u(x,t)=\left[\sum_{i=0}^{2}\tau_i\left(\alpha_2+\frac{(\alpha_2-\alpha_1)}{exp\left[B_3\left(x-\frac{q^2\zeta_0\tau_1\left(b\tau_1^2+a(q+1)(q+2)\tau_2\right)}{(q+1)(q+2)\tau_2(q^2\zeta_0\tau_1+4c\xi_1\tau_2)}t\right)\right]-1}\right)^i\right]^{\frac{1}{q}}, \tag{3.75}$$

$$u(x,t)=\left[\sum_{i=0}^{2}\tau_i\left(\alpha_1+\frac{(\alpha_1-\alpha_2)}{exp\left[B_3\left(x-\frac{q^2\zeta_0\tau_1\left(b\tau_1^2+a(q+1)(q+2)\tau_2\right)}{(q+1)(q+2)\tau_2(q^2\zeta_0\tau_1+4c\xi_1\tau_2)}t\right)\right]-1}\right)^i\right]^{\frac{1}{q}}, \tag{3.76}$$

$$u(x,t)=\left[\sum_{i=0}^{2}\tau_i\left(\alpha_1-\frac{2(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}{2\alpha_1-\alpha_2-\alpha_3+(\alpha_3-\alpha_2)cosh\left[C(x-\lambda t)\right]}\right)^i\right]^{\frac{1}{q}}, \tag{3.77}$$

$$u(x,t)=\left[\sum_{i=0}^{2}\tau_i\left(\alpha_2+\frac{(\alpha_1-\alpha_2)(\alpha_4-\alpha_2)}{\alpha_4-\alpha_2+(\alpha_1-\alpha_4)sin^2(\varphi,l)}\right)^i\right]^{\frac{1}{q}}, \tag{3.78}$$

where
$A_{12}=\sqrt{\frac{\zeta_0 2c\tau_1\left(b\tau_1^2+a(q+1)(q+2)\tau_2\right)}{b\tau_2^2(q^2\zeta_0\tau_1+4c\xi_1\tau_2)}}, B_3=\frac{k(\alpha_1-\alpha_2)}{A_{12}}, C=\frac{k\sqrt{(\alpha_1-\alpha_2)(\alpha_1-\alpha_3)}}{A_{12}}$,
$\varphi=\frac{k\sqrt{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}}{2A_{12}}(x-\lambda t), l^2=\frac{(\alpha_2-\alpha_3)(\alpha_1-\alpha_4)}{(\alpha_1-\alpha_3)(\alpha_2-\alpha_4)}, \lambda=\frac{q^2\zeta_0\tau_1\left(b\tau_1^2+a(q+1)(q+2)\tau_2\right)}{(q+1)(q+2)\tau_2(q^2\zeta_0\tau_1+4c\xi_1\tau_2)}$.
Here, A is the amplitude of the soliton, while λ is the velocity and B and C are the inverse width of the solitons.
Remark 2. The solutions of Eq. (1.2) were obtained by the way of using ETEM. They were controlled in Mathematica 12. We have obtained the similar solution with the solution Eq. (17) in Osman et al. (2018), with the solution Eq. (6) in Khalique (2013) and with the solution Eq. (58) in Aminikhah et al. (2015) in this work with the solution Eq. (3.53). Moreover, we have obtained the similar solution with the solution Eq. (57) in Aminikhah et al. (2015) in this work with the solution Eq. (3.54). In addition, other solutions of Eq. (1.2) are new.

4. Conclusion

Travelling wave solutions of these equations were found by applying ETEM to (2+1) dimensional NETLE and BBMP equations. It's should be note that ETEM ensures strong mathematical means for obtaining the exact solutions of these equations and this method is very effective in seeking novel solutions such as soliton solutions, rational, Jacobi elliptic, periodic wave solutions and hyperbolic function solutions.

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Author's contributions

The author contributed to the writing of this paper. The author read and approved the final manuscript.

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