Research Article

New exact solution for (2+1) and (3+1) dimensional nonlinear partial differential equations

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Abstract

In this paper, dark soliton solutions have been obtained for the (2+1)-dimensional reaction-diffusion equation, the (3+1)-dimensional generalized B-type Kadomtsev-Petviashvili (gBKP) equation and the (3+1)-dimensional Boiti-Leon-Manna-Pempinelli (BLMP) equation using the solitary wave ansatz. Ansatz approach is utilized to carry out this integration. The constraint relations for each of the equations are given for the existence of dark soliton solutions.

Keywords

Ansatz method, dark soliton solutions, exact solution

1. INTRODUCTION

 Nonlinear partial differential equations (NLPDEs) that are studied in the fields of physics and applied mathematics there are significant advances and across the globe many aspects of NLPDEs that are studied by various scientists. Some of them are the integrability aspects, symmetry issues, conservation laws. There has been quite thoroughly understood is the integrability aspect of the governing equations of the solitary waves in (2+1)-dimensional as well as in (3+1)-dimensional equations. For a long time, there are many new results related this aspects that are published in several area [1-6]. More commonly used methods of integrability are the He's variational iteration method, the hirota bilinear method, the homotopy perturbation

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method, the first integral method, the modified tanh--coth method, the modified sine-cosine method, the ansatz method and many others [7-12]. Among the methods mentioned above, the ansatz method is one of the most efficient method to determine solutions of NLPDEs [13-22]. It was originally developed by Anjan Biswas [23-25].

In nonlinear dynamical systems, propagation of nonlinear waves has been a fundamental objects of nature. Nonlinear waves emerge to in a great array of contexts such as, solid state physics, hydrodynamics, nuclear physics, plasmas, nonlinear optics and many other nonlinear phenomena. In this paper, we apply the ansatz method to obtain dark soliton solutions of the following (2+1)-dimensional reaction-diffusion equation, the (3+1)-dimensional gBKP equation and the (3+1)-dimensional BLMP equation. Dark soliton solutions is one of the fastest growing research fields in the context of wave phenomena [26, 27].

2. THE (2+1)-DIMENSIONAL REACTION-DIFFUSION EQUATION

Let us consider the following reaction-diffusion equation in two dimensions,

$$(\psi_t - D\psi_{xx} + \alpha\psi^3 + \beta\psi^2 + \gamma\psi)_x + k\psi_{yy} = 0, \tag{2.1}$$

where D, α, β, γ and k are real constants, D is also called diffusion coefficient. In [28], Yun-Quan and Jun investigated the explicit analytical solutions for this equation (2.1) by using the first integral method.

For k=0, equation (2.1) is a reaction-diffusion equation arising in chemical reaction or ecology and other fields of physics. The choice D = 1, $\beta = 0$ and $\gamma = -\alpha$ leads Eq. (2.1) to the two-dimensional Chaffee-Infante equation,

$$(\psi_t - \psi_{xx} + \alpha\psi^3 - \alpha\psi)_x + k\psi_{yy} = 0, \tag{2.2}$$

the choice D = 1, $\gamma = 0$ and $\alpha = -\beta = 1$ leads Eq. (2.1) to the two-dimensional Huxley equation,

$$(\psi_t - \psi_{xx} + \psi^3 - \psi)_x + k\psi_{yy} = 0, \tag{2.3}$$

and if taking D = 1, $\alpha = 1$, $\beta = -(a+1)$ and $\gamma = a$ in Eq. (2.1), we can obtain the following two-dimensional Fitzhugh--Nagumo equation:

$$(\psi_t - \psi_{xx} + \psi^3 - (a+1)\psi^2 + a\psi)_x + k\psi_{yy} = 0. \tag{2.4}$$

In order to get its solution, the following ansatz is assumed [24, 29],

$$\psi(x,t) = \lambda\tanh^p\tau, \tag{2.5}$$

and

$$\tau = ax + by - vt, \tag{2.6}$$

where the λ, a and b are the free parameters, and v is the velocity of the soliton. From Eqs. (2.5) and (2.6), we have:

$$-p\lambda va\{(p+1)\tanh^{p+2}\tau - 2p\tanh^p\tau + (p-1)\tanh^{p-2}\tau\}$$

$$- D\lambda pa^3\{(p-1)(p-2)(\tanh^{p-3}\tau - \tanh^{p-1}\}\tau)$$

$$+ (p+1)(p+2)(\tanh^{p+1}\tau - \tanh^{p+3}\tau) + 2p^2(\tanh^{p+1}\tau - \tanh^{p-1}\tau)\} \quad (2.7)$$

$$+3\alpha\lambda^3 pa\{\tanh^{3p-1}\tau - \tanh^{3p+1}\tau\}+2\beta\lambda^2 pa\{\tanh^{2p-1}\tau - \tanh^{2p+1}\tau\}$$

$$+\gamma\lambda pa\{\tanh^{p-1}\tau - \tanh^{p+1}\tau\}+k\lambda pb^2\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau+(p+1)\tanh^{p+2}\tau\} = 0.$$

By equating the highest exponents of $\tanh^{3p+1}\tau$ and $\tanh^{p+3}\tau$ terms in Eq (2.7), we obtain

$$3p+1 = p+3, \quad (2.8)$$
$$p = 1.$$

The same value of p can yield when the exponents pairs 2p+1 and p+2, 2p-1 and p are equate with each other. Furthermore, set the coefficients of the linearly independent terms to zero. By setting the corresponding coefficients of $\tanh^{3p+1}\tau$ and $\tanh^{p+3}\tau$ terms to zero we get

$$D\lambda p(p+1)(p+2)a^3 - 3\alpha\lambda^3 pa = 0, \quad (2.9)$$

after some calculations we obtain

$$\lambda = \pm a\sqrt{\frac{(p+1)(p+2)D}{3\alpha}}, \quad (2.10)$$

which implies that is necessary to have $D\alpha>0$.

Setting the coefficient of $\tanh^{p-1}\tau$ terms in Eq. (2.7), to one obtains

$$2p^3 D\lambda a^3 + \gamma\lambda pa = 0, \quad (2.11)$$

which gives

$$a = \pm\sqrt{\frac{\gamma}{2p^2 D}} \quad . \quad (2.12)$$

Thus from (2.12) it is possible to conclude that the topological solitons will exist for $\gamma D<0$. Again from (2.7), matching the exponents of $\tanh^{2p+1}\tau$ and $\tanh^{p+2}\tau$ to zero yields

$$-p(p+1)\lambda va - 2\beta\lambda^2 pa + k\lambda p(p+1)b^2 = 0, \quad (2.13)$$

we obtain

$$v = \frac{2\beta\lambda a - k(p+1)b^2}{(p+1)a} \quad . \quad (2.14)$$

Hence, we obtain the topological soliton solution for the equation (2.1) when we substitude (2.8), (2.10), (2.12), (2.14) in (2.5) as

$$\psi(x,y,t) = \lambda\tanh(ax + by - vt), \quad (2.15)$$

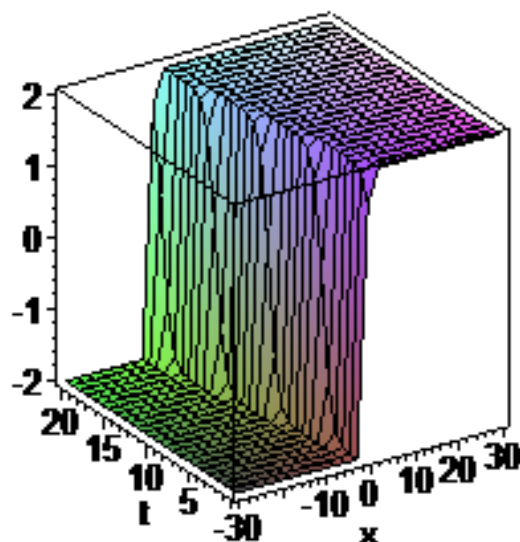which exist provided that $\gamma D<0$ and $D\alpha>0$.

Figure 1. Shock wave $\psi(x,1,t)$ with $\gamma = -8$, $\alpha = 2$, $\beta = 1$, $k = 2$, $D = 1$, $b = 1$.

3. THE (3+1)-DIMENSIONAL GENERALİZED B-TYPE KADOMTSEV PETVIASHVILI (gBKP) EQUATION

Now we consider the (3+1)-dimensional gBKP equation [30],

$$\psi_{xxxy} + \alpha(\psi_x \psi_y)_x + \beta(\psi_x + \psi_y + \psi_z)_t + \gamma(\psi_{xx} + \psi_{zz}) = 0, \tag{3.1}$$

where α, β and γ are real-valued constant. This is a nonlinear wave equation in three spatial (x,y,z) and one temporal coordinate (t). Khalique and Abudiab studied the generalized BKP equation and derived some exact solutions using the multiple-exp function and simplest equation methods. Also, the conservation laws for the BKP equation are constructed by using the multiplier method by them. In [31], a new form of the (3+1)-dimensional BKP equation given by (3.1) was investigated and it was shown, using the simplified form of the Hirota method, that one- and two-soliton solutions exist for (3.1). Also, specific constraints were developed that guarantee the existence of multiple soliton solutions for (3.1). The (3+1)-dimensional nonlinear gBKP equation

$$\psi_{yt} - \psi_{xxxy} - 3(\psi_x \psi_y)_x + 3(\psi_{xx} + \psi_{zz}) = 0, \tag{3.2}$$

was studied in [32-35] by different approaches.

The other variant of a generalized (3+1)-dimensional BKP equation given by

$$\psi_{xxxy} - 3(\psi_x \psi_y)_x + \psi_{tx} + \psi_{ty} - \psi_{zz} = 0, \tag{3.3}$$

In [36], Ma and Abdeljabbar has computed a bilinear Bäcklund transformation for Eq. (3.3) and in [37], Wronskian and Grammian formulations are established for this equation. Ma and Zhu

[38], applied the multiple exp-function algorithm to this equation. In [39],Wazwaz studied three variants of a generalized (3+1)-dimensional BKP equation in the form

$$\psi_{zt} - \psi_{xxxy} - 3(\psi_x \psi_y)_x + 3\psi_{xx} = 0, \tag{3.4}$$

$$\psi_{yt} - \psi_{xxxz} - 3(\psi_x \psi_z)_x + 3\psi_{yy} = 0,$$

$$\psi_{xt} - \psi_{xxxy} - 3(\psi_x \psi_y)_x + 3\psi_{zz} = 0,$$

and multiple-front wave solutions of Eq. (3.4) have been obtained by using the simplified form of the Hirota's method. Also, in [40] Ma and Fan studied the first (3+1)-dimensional gBKP equation and presenting its particular exact multiple wave solutions.

In order to construct dark soliton solutions, we use

$$\psi(x,y,z,t) = \lambda \tanh^p \tau, \tag{3.5}$$

and

$$\tau = ax + by + cz - vt, \tag{3.6}$$

where λ, a, b and c are free parameters and v is the velocity of the soliton. From Eq. (3.5) and (3.6), we have

$$\lambda pa^3 b\{(p-1)(p-2)(p-3)\tanh^{p-4}\tau - 4(p-1)(p^2-2p+2)\tanh^{p-2}\tau + 2p(3p^2+5)\tanh^p\tau$$

$$- 4(p+1)(p^2+2p+2)\tanh^{p+2}\tau + (p+1)(p+2)(p+3)\tanh^{p+4}\tau\} + \alpha\lambda^2 p^2 a^2 b\{(2p-2)\tanh^{\wedge}\{2p-3\}\tau$$

$$- (6p-2)\tanh^{2p-1}\tau + (6p+2)\tanh^{2p+1}\tau - (2p+2)\tanh^{2p+3}\tau\} \tag{3.7}$$

$$- \beta p\lambda v(a+b+c)\{(p+1)\tanh^{p+2}\tau - 2p\tanh^p\tau + (p-1)\tanh^{p-2}\tau\}$$

$$+ \gamma p\lambda(a^2+c^2)\{(p-1)\tanh^{p-2}\tau - 2p\tanh^p\tau + (p+1)\tanh^{p+2}\tau\} = 0.$$

When we match the exponents of $\tanh^{2p+3}\tau$ and $\tanh^{p+4}\tau$ terms in Eq (3.7), we obtain

$$2p+3 = p+4, \tag{3.8}$$

$$p = 1. \tag{3.9}$$

By setting the coefficients of $\tanh^{2p+3}\tau$ and $\tanh^{p+4}\tau$ terms to zero we get

$$\lambda p(p+1)(p+2)(p+3)a^3 b - 2\alpha\lambda^2 p^2(p+1)a^2 b = 0, \tag{3.10}$$

so that

$$\lambda = \frac{(p+2)(p+3)a}{2\alpha p}. \tag{3.11}$$

Again, by equating the coefficients of $\tanh^{2p+1}\tau$ and $\tanh^{p+2}\tau$ terms to zero

$$-4\lambda pa^3 b(p+1)(p^2+2p+2) + \alpha\lambda^2 p^2(6p+2)a^2 b - \beta p(p+1)\lambda v(a+b+c) + \gamma p(p+1)\lambda(a^2+c^2) = 0. \tag{3.12}$$

Solving the equation (3.12) with by using (3.11)

$$v = \frac{-4(p+1)(p^2+2p+2)a^3b + \alpha(3p+1)(p+2)(p+3)a^3b + \gamma(p+1)(a^2+c^2)}{p(p+1)\beta(a+b+c)}. \tag{3.13}$$

The relations (3.11) and (3.13) introduce the restrictions given by

$$\alpha \neq 0, \tag{3.14}$$

$$\beta(a+b+c) \neq 0.$$

Dark soliton solution for the equation (3.1) is given by

$$\psi(x,y,z,t) = \lambda \tanh(ax + by + cz - vt), \tag{3.15}$$

where the free parameter λ is given by (3.11) and velocity of the solitons v is given in (3.13). In (3.13) v is dependent on the other free parameters a, b and c.
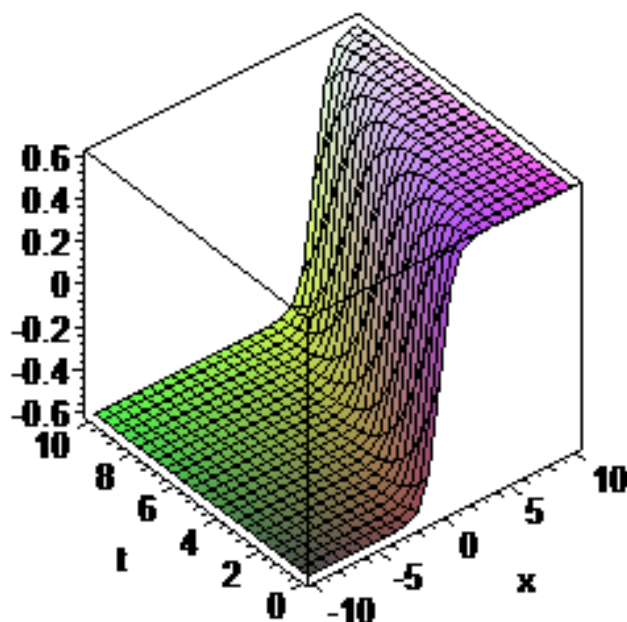

Figure 2. Shock wave Eq.(3.1) with $\gamma = 1$, $\alpha = 10$, $\beta = 10$, a = 1, b= 10, c = 10, p = 1

4. THE (3+1)-DIMENSIONAL BOITI-LEON-MANNA-PEMPINELLI EQUATION

For initial data decaying rapidly at infinity, Boiti, Leon, Manna and Pempinelli have developed an inverse scattering scheme to solve the Cauchy problem of a (2+1)-dimensional generalization of the shallow water wave equations:

$$\psi_{yt} + \psi_{xxxy} - 3\psi_{xx}\psi_y - 3\psi_x\psi_{xy} = 0, \tag{4.1}$$

Eq. (4.1), it is also known (2+1)-dimensional BLMP equation has been studied in detail by many scientists. For example, Song-Hua and Jian-Ping obtained various exact solutions which

include solitary wave solution and by selecting appropriate functions, they investigated some novel localized excitations such as multi dromion-solitoffs and fractal-solitons. Eq. (4.1) has been considered as a (2+1)-dimensional generalization of the shallow water wave equations in [41]. Painlevé analysis, Lax pair and some exact solutions of Eq. (4.1) have been given in [42] and some soliton-like solutions have been obtained through the symbolic-computation-based method in [43]. Gao an Tian studied this equation and derived some new soliton-like solutions using the symbolic computation with the generalized tanh method [44].

Now, we introduce an extension to Eq. (4.1) by adding the last three terms with y replaced by z. This extension enables us to establish a BLMP equation as

$$\psi_{yt} + \psi_{zt} + \psi_{xxxy} + \psi_{xxxz} - 3\psi_x(\psi_{xy} + \psi_{xz}) - 3\psi_{xx}\psi_y - 3\psi_{xx}\psi_z = 0, \tag{4.2}$$

In [45], Darvishi et al. have obtained single-wave, double-wave and multi-wave solutions of Eq. (4.2) by the multiple exp-function method. Similarly, in order to construct dark soliton solutions for Eq. (4.2), we use an ansatz solution of the form (3.5) and (3.6). From Eq. (3.5) and (3.6), we have

$$-p\lambda v(b+c)\{(p+1)\tanh^{p+2}\tau - 2p\tanh^p\tau + (p-1)\tanh^{p-2}\tau\} + \lambda pa^3(b+c)\{(p-1)(p-2)(p-3)\tanh^{p-4}\tau$$

$$-4(p-1)(p^2-2p+2)\tanh^{p-2}\tau + 2p(3p^2+5)\tanh^p\tau - 4(p+1)(p^2+2p+2)\tanh^{p+2}\tau$$

$$+(p+1)(p+2)(p+3)\tanh^{p+4}\tau\} - 6\lambda^2 p^2 a^2(b+c)\{(p-1)\tanh^{2p-3}\tau - (3p-1)\tanh^{2p-1}\tau \tag{4.3}$$

$$+(3p+1)\tanh^{2p+1}\tau - (p+1)\tanh^{2p+3}\tau\} = 0.$$

By equating the exponents of $\tanh^{2p+3}\tau$ and $\tanh^{p+4}\tau$ terms in Eq (4.3),

$$2p+3 = p+4, \tag{4.4}$$

we get

$$p = 1. \tag{4.5}$$

Furthermore, set the coefficients of the linearly independent terms to zero.

$$\lambda p(p+1)(p+2)(p+3)a^3(b+c) + 6\lambda^2 p^2(p+1)a^2(b+c) = 0, \tag{4.6}$$

$$-p(p+1)\lambda v(b+c) - 4\lambda a^3(b+c)p(p+1)(p^2+2p+2) - 6\lambda^2 p^2(3p+1)a^2(b+c) = 0. \tag{4.7}$$

Solving the above system of equations and also set p=1, then it can be written

$$\lambda = -2a, \tag{4.8}$$

$$v = 4a^3. \tag{4.9}$$

Further, the Eqs. (4.6) and (4.7) imply that the solution will exist for b+c≠0. Therefore the dark soliton solution or topological 1-soliton solution of (3+1)-dimensional BLMP equation (4.2) is given by

$$\psi(x,y,z,t) = -2a.\tanh(ax + by + cz - 4a^3t), \qquad (4.10)$$

Remark: Solution (4.10) is also solution of the (2+1)-dimensional BLMP equation.
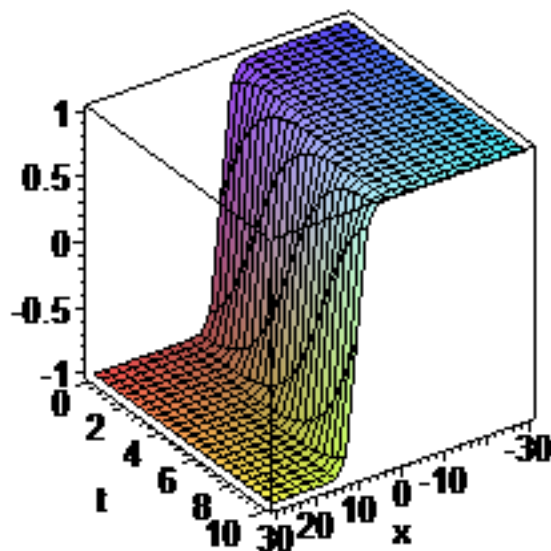

Figure 3. Dark soliton solution Eq.(4.2) with a = 1, p = 1

5. CONCLUSION

In this paper; dark soliton solutions of the (2+1)-dimensional reaction-diffusion, the (3+1)-dimensional generalized B-type Kadomtsev-Petviashvili (BKP) and the (3+1)-dimensional BLMP equations are obtained by ansatz method. The ansatz method was employed to carry out the integration. The dark soliton solutions also introduced several constraint conditions that must remain valid in order for these solutions to exist.

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